PART I

PUBLIC PROCUREMENT UNDER UNCERTAINTY AND COMPLEX ENVIRONMENTS

FINDING OPTIMAL PROCUREMENT TRATEGIES UNDER RISKS

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ABSTRACT. Choosing the best procurement strategy for allocating contracts by any public administration under risks is the challenge that has motivated this research. For any set of unfavorable events that may occur in the course of both allocating and implementing contracts, quantitative techniques for finding the best procurement strategies are developed, provided some statistical data for estimating the chances of the events to occur is available. When the administration has financial resources to spend to reduce the chances of unfavorable events to occur, the best administration's procurement strategy—consisting of choosing an optimal combination of the type of the contract with the type of a tender procedure to award the contract—can be found by mathematical programming techniques. Also, a mechanism for one-step sealed bid auctions reducing the risks associated with both the allocation and implementation of the awarded contracts as a result of applying the mechanism is discussed.

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MAXIMIZING THE CHANCES OF SUCCESSFULLY IMPLEMENTING A CONTRACT BY OPTIMALLY ALLOCATING AVAILABLE FINANCIAL RESOURCES

In allocating state contracts in many countries, for instance, in Russia, at any of the three levels— state, regional, and municipal—the public administration responsible for the contract allocation should take into account the risks of unfavorable events that may occur in the course of both allocating (placing) orders and contact implementation.

A set of the strategies that the administration can choose attempting to reduce the risks of the low quality of the contract implementation, as well as the risks of the failure to fulfill the contract obligations by the contractors involved, includes, but is not limited to cost contract, cost sharing contract, cost-plus-incentive-fee contract, cost-plus-award fee contract, cost-plus-fixed-fee contract.

It seems convenient to depict information about the unfavorable events that can affect the allocation and the implementation of a state order in the form of a matrix. The rows of this matrix correspond to the above strategies to which the administration can adhere in placing the order, whereas the matrix columns correspond to the unfavorable events that may occur in the course of both the placement and the implementation of the order. Thus, each element of the matrix associated with a particular order is the probability of a particular unfavorable event to occur under each particular administration strategy. This strategy is a combination of the type of a contract for implementing this order and the type of a tender procedure in the framework of which the administration can tender this contract). .

One should bear in mind that though finding even the estimates of the above probabilities is a difficult task, one cannot talk about any quantitative methods of the “risk management” in the absence of such estimates. It is further assumed that certain quantitative information on the mentioned probabilities is known.

Any public administration placing state orders can find itself in two financial situations: a) it has financial resources to spend to reduce the probabilities of unfavorable events to occur, and b) it does not have such financial resources.

Let

r_i be a strategy that the administration can choose in allocating a particular order,

C_j be an unfavorable event that may occur in the course of allocating and implementing a particular order and that presents an obstacle in successfully implementing the order, ,

b be the amount of financial resources at the public administration's disposal, which can be

used to reduce the chances (probabilities) of unfavorable events to occur,

$P(A)$ be the probability of the event A,

Q be the event consisting of the successful implementation of the contract,

$\overline{A}$ be the negation of the event A.

Assuming that all the unfavorable events are pair-wise independent, one can easily be certain that the following relations hold:

$$. P(Q) = \prod_{j=1}^{n} P\left(\overline{C}_j\right)$$

In situation a), to calculate the probability $P(Q)$ for each particular strategy r_i, $i \in \overline{1,m}$ and to choose the strategy that maximizes this probability is the best the administration can do, which means that the administration should calculate the number

$$\max_{i\in\overline{1,m}} P_i(Q) = \max_{i\in\overline{1,m}} P_i\left(\prod_{j=1}^{n} \overline{C}_j\right) = \max_{i\in\overline{1,m}} \prod_{j=1}^{n} P_i\left(\overline{C}_j\right), \quad (1)$$

where

$P_i(Q)$ is the probability of successfully implementing the contract by a contractor under administration strategy i, i.e., under choosing a combination of the procedure of allocating a contract and the type of the contract that correspond to row i of the above matrix,

$P_i(\overline{C}_j)$ is the probability that unfavorable event j will not occur if the administration chooses strategy i, $i \in \overline{1,m}$, $j \in \overline{1,n}$,

In situation b), one should find how the probabilities $P_i\left(\overline{C}_j\right)$ depend on the amount of financial resources that the administration can afford to spend for reducing these probabilities, assuming that all the probabilities $P_i\left(C_j\right)$, $i \in \overline{1,m}$, $j \in \overline{1,n}$, can be reduced on account of certain activities, each requiring financing.

Let x_j, $j \in \overline{1,m}$ be the amount of financing that the administration can afford to spend in an attempt to reduce the risk of unfavorable event j, $j \in \overline{1,n}$, to occur. It seems natural to assume that the more the x_j, the more generally the probabilities $P_i\left(\overline{C}_j\right)$ [1]. It is also natural to assume that there is a certain threshold θ_{ij} such that spending amounts of financial resources for reducing the probability $P_i\left(\overline{C}_j\right)$ that exceed θ_{ij} at least cannot reduce this probability comparing with the value of the probability at $x_j = \theta_{ij}$. This phenomenon is well known in similar situations associated with the perception of advertising messages for goods and services [1, 2].

Further, let $P_{ij}(x_{ij}) = (P_i(C_j))(x_{ij})$ be the functions reflecting regularities describing how the probabilities of unfavorable events to occur depend on the financial resources spent on their reduction. Under the assumptions made, the functions $P_{ij}(x_{ij})$ are decreasing (at least non-increasing) monotone functions on the segment $[0, \theta_{ij}]$ and non-decreasing on the segment (θ_{ij}, ∞).

It is natural to assume that in situations that any administration may face, the inequality

$$b < \min_{i \in \overline{1,m}} \sum_{j=1}^{n} \theta_{ij}$$

holds.

The problem of optimally allocating financial resources to be spent for reducing the probabilities of unfavorable events associated with allocating state orders when the administration chooses particular strategy i, $i \in \overline{1,m}$ can be written as follows:

$$\begin{aligned} &\sum_{j=1}^{n} x_{ij} \le b, \\ &0 \le x_{ij} \le \theta_{ij},\ j \in \overline{1,n}, \qquad (2) \\ &\prod_{j=1}^{n} \left(1 - P_{ij}(x_{ij})\right) \to \max \end{aligned}$$

This problem is a non-linear programming problem that can be solved by well-developed non-linear programming techniques for particular types of functions $P_{ij}(x_{ij})$ that may describe the probabilities. Several examples of such particular functions $P_{ij}(x_{ij})$ are considered in [1], and it turns out that for the functions of a particular type, solution techniques for solving problem (2) that are more effective than general non-linear programming techniques can be developed.

Thus, the very possibility to manage the risks in allocating orders and implementing contracts depends on the available financial resources that can be spent for conducting activities that have the potential of reducing these risks. If such resources are absent, one should solve problem (1), whereas if the resources are available, one should solve problems (2) and find a number i^* for which the equality

$$\max_{(x_{i^*1},\dots,x_{i^*n})\in M_{i^*}} \prod_{j=1}^{n}\left(1-P_{i^*j}(x^*_{i^*j})\right) = \max_{i\in\overline{1,m}} \max_{(x_{i1},\dots,x_{in})\in M_i} \prod_{j=1}^{n}\left(1-P_{ij}(x_{ij})\right)$$

holds, where M_i is a set of feasible solutions to problem (2)

for a particular $i \in \overline{1,m}$.

Remark 1. Currently, one can talk about two major directions of managing risks in allocating state orders. Pointing out the most complete set of unfavorable events that one would like to avoid in the process of allocating orders and implementing contracts, along with a brief description of consequences that such events may cause, constitutes the first direction. There is a number of publications in which such unfavorable events are listed [3].

Developing and analyzing particular scenarios of the occurrence of unfavorable events in public procurement represents the second direction. Publications on this topic usually are done by the companies that are successful in using such scenarios and are, in fact, certain advertizing materials that do not specify how the scenarios have been used, which represents a commercial secret. For instance, the experience of Hewlett-Packard in organizing the Procurement Risk Management Group for developing and analyzing possible scenarios of occurring unfavorable events that may take place in the course of public procurement is reflected in [4]. This publication is a standard publication of the kind, and the company’s experience can be used in allocating state orders and managing awarded contracts under risk.

The approach proposed in the present article is used for forming the structure of the matrix “administration strategies—unfavorable events” and should be considered as the one in the second direction. Indeed, one can view calculating probabilities of successfully implementing contracts under a particular administration strategy (a combination of a variant of conducting a tender and a type of a contact to be awarded to the winner of the tender as a version of a scenario associated with a particular allocation and implementation of the contract.

Remark 2. As mentioned earlier, for particular types of the functions $P_{ij}\left(x_{ij}\right)$ for problem (2) one can develop methods that are more effective that existing general techniques for solving mathematical programming problems. For instance, if the functions $P_{ij}\left(x_{ij}\right)$ look as follows:

$$P_{ij}\left(x_{ij}\right)=\begin{cases} p_{ij}^{0}\left(1-x_{ij}^{\alpha_{ij}}\right),\ 0\le x_{ij}\le\theta_{ij} \\ p_{ij}^{0}\left(1-\theta_{ij}^{\ \alpha_{ij}}\right), x_{ij}>\theta_{ij}\ ,\ i\in\overline{1,m},\ j\in\overline{1,n}, \end{cases}$$

where the inequalities $\alpha_{ij}<1$ and $x_{ij}^{\alpha_{ij}}<1$ for $0\le x_{ij}\le\theta_{ij}$ hold, problem (2) for analyzing administrative strategy i takes the form

$$\prod_{j=1}^{n}\left(q_{ij}^{0}+p_{ij}^{0}x_{ij}^{\alpha_{ij}}\right)\rightarrow\max$$

$$\sum_{j=1}^{n}x_{ij}\le b$$

$$0\le x_{ij}\le\theta_{ij}\ ,\ \ j\in\overline{1,n},$$

where $q_{ij}^{0}=1-p_{ij}^{0}$.

In this problem, all the constraints and the goal function are functions-posynomials, which allows one to use geometric programming ideas fro solving this problem [5].

THE IDEA OF A MECHANISM (RULE) FOR DETERMINING THE WINNER IN A SEALED BID AUCTION FOR A CONTRACT

Throughout the rest of the article, only tender procedures in the form of sealed-bid auctions are the subject of consideration.

It is clear that if the auction organizer knew that all the auction participants are reputable, experienced companies, capable of completing the contract (the subject of the auction) timely and in line with the quality requirements, the risks of the auction organizer in both allocating and implementing the contract would be substantially reduced. So designing economic mechanisms for conducting the auctions, i.e., designing the rules of determining the auction winner that would reduce both mentioned types of risks, seems important. One such mechanism was first proposed in [6] and further developed in [7]. It turns out that the proposed mechanism does serve two particular goals of the auction organizers—makes it attractive for both the auction organizer and for the auction participants.

The mechanism was designed to deal with the problem of allocating a contract for implementing work (or a set of works), for instance, in public procurement among a finite number of potential participants consisting of four groups of companies or businesses. The first group is formed by reputable, experienced participants (from the organizer's viewpoint) in each of which the organizer has confidence regarding the quality of its performance and its ability to have the wok done timely. The second group consists of companies that do not have appropriate experience of performing the job, being the subject of the contract, do not properly evaluate real expenditures associated with implementing the contract, and care about the money that they can eventually get (if they win the contract) more than about the quality of implementing the contract and, sometimes, even about implementing it at all. Companies willing to win the contract and ready to do whatever it takes to undermine positions of their competitors in the market by not letting them win the contract form the third group. Finally, the fourth group consists of companies that would like to establish their presence in the market, are ready to bear additional expenses, for instance, associated with hiring external experts, and are ready to receive the contract at a price that may be lower (even substantially) than their expenditures associated with implementing the contract.

If the rules for determining the winner are such that "the lowest price" always wins, then companies from the first group find themselves in a disadvantageous situation, since they cannot afford to lower the prices to be submitted by them and consequently do not have a chance to win. So the "lowest price" mechanism, serving only one of at least two equally important goals—the price and the quality of implementing a contract—may be unacceptable to the auction organizers.

A set of rules aimed at making the submission of dumping prices unprofitable for the participants of the bid while encouraging each participant to evaluate the chances of other potential participants, especially those from the first group, to submit their prices within a certain range was first proposed in [6]. The idea underlying the rules is not to declare a certain maximal (though not necessarily a reserved) price of the bid that the organizer does not want to exceed, but rather to guarantee to the winner the contract price to be within a segment between a certain high percentage of this (unknown to the participants) maximal price and the maximal price itself. That is, if x is the maximal price, the winning price is guaranteed to be within the segment $[kx, x]$, where $k < 1$, and the winner is determined as follows:

a) if all the submitted prices do not exceed kx, then the participant who has submitted the price that is either the closest to kx (among all the submitted prices) or coincides with kx is declared the winner, and the winning price is kx,

b) if all the submitted prices are not lower than kx, then a participant who has submitted the price that is either the closest to kx (among all the submitted prices) or coincides with kx is declared the winner, and the winning price is the price submitted by the winner,

c) if some of the participants submitted the prices that do not exceed *kx*, whereas the others submitted the prices that are not lower than *kx*, rule a) applies, and the winning price is *kx,*

d) if several participants submitted the same winning price, the winner is determined by an additional procedure, whereas the winning price is determined by either rule a) or rule b),

e) if all the submitted prices exceed *x*, the sealed-bid auction is considered as failed.

It turns out that based upon a probabilistic evaluation of the chances of the participants from the first group to submit prices within a certain range and the maximal price that the organizer can afford to pay for the contract being the subject of the auction, the bid organizer can determine both *x* and *k* that minimize the probability *P*(*T*) of the winning price to exceed *kx* [6], [7].

Let

n be the number of participants of the sealed bid ceiling auction in which particular work or (a set of works) is (are) the subject of the auction,

T be the event consisting of that the contract will be given to the winning contractor at the price that exceeds *kx*,

A_i be the event consisting of that auction participant *i* submits the price for the contract that does not exceed *kx*, $i \in \overline{1,n}$,

C_i be the event consisting of that auction participant *i* submits the price that exceeds *x* .

Further, let h_i be the price for the contract that the auction organizer believes that participant *i* is likely to submit as his (her, its) bid. If the analysis conducted by the auction organizer shows that h_i can vary within the segment $\underline{h}_i^f \le h_i \le \overline{h}_i^f$, $i \in \overline{1,n}$, whereas the auction organizer does not have any information about the preferences that auction participant *i* may have in choosing a particular value of h_i from this segment, it is natural to assume that h_i is a continuous uniformly distributed random variable with the probability density $p(h_i)$, where

$$p(h_i) = \begin{cases} 0, \text{if } h_i < \underline{h}_i^f \\ 1/(\overline{h}_i^f - \underline{h}_i^f), \text{if } \underline{h}_i^f < h_i < \overline{h}_i^f \\ 0, \text{if } h_i > \overline{h}_i^f \end{cases}$$

Proceeding from these assumptions, the auction organizer can choose both parameters *k* and *x* and estimate the probability of the event *T*. To this end, the auction organizer should consider how the function *P*(*T*) –the probability of the event *T* –will look under different mutual location of the segments [*kx*,*x*] and $[\underline{h}_i^f, \overline{h}_i^f]$, $i \in \overline{1,n}$ in the set of real numbers. As shown in [6], it is sufficient to consider the following five cases of their mutual location for auction participant *i*

a) $kx < x \leq \underline{h}_i^f < \overline{h}_i^f$,

b) $kx < \underline{h}_i^f < x < \overline{h}_i^f$,

c) $\underline{h}_i^f < kx < x < \overline{h}_i^f$, (3)

d) $\underline{h}_i^f < kx < \overline{h}_i^f < x$,

e) $\underline{h}_i^f < \overline{h}_i^f < kx < x$.

If the minimal price that auction participant *i* submits as its bid exceeds *kx* –the price for the contract desirable for the auction organizer—then the probability of the event A_i equals zero, since A_i becomes an impossible event so that in cases a) и b) $P(A_i) = 0$. In case e), the event A_i becomes a certain event so that $P(A_i) = 1$. In both remaining cases, the probability of the event A_i is a linear function of *kx*

$$P(A_i) = \frac{kx - \underline{h}_i^f}{\overline{h}_i^f - \underline{h}_i^f}.$$

Similar considerations let one be certain that in case a), C_i is a certain event, whereas this event becomes an impossible event in cases d) and e). Thus, $P(C_i) = 1$ in case a) and $P(C_i) = 0$ in cases d) и e), whereas in both remaining cases b) and c), the probability of the event C_i is a linear function of *h*,

$$P(C_i) = \frac{\overline{h}_i^f - x}{\overline{h}_i^f - \underline{h}_i^f}$$

It is obvious that if case e) takes place for at least one of the auction participants, the event *T* becomes an impossible event, i.e., *P*(*T*)=0, so only cases a)-d) present interest for further consideration.

One can easily be certain that if cases a)-c) take place for all the auction participants, the probability that the contract will be given to the winner at the price exceeding *kx* is described by the function

$$P(T) = \prod_{i=1}^{n} \min\left(1, \frac{\overline{h}_i^f - kx}{\overline{h}_i^f - \underline{h}_i^f}\right) - \prod_{i=1}^{n} \min\left(1, \frac{\overline{h}_i^f - x}{\overline{h}_i^f - \underline{h}_i^f}\right), \qquad (4)$$

whereas if case d) takes place for at least one auction participant, this probability is described by the function

$$P(T) = \prod_{i=1}^{n} \min\left(1, \frac{\overline{h}_i^f - kx}{\overline{h}_i^f - \underline{h}_i^f}\right). \qquad (5)$$

The function *P*(*T*) is that of variables *k* and *x* for which obvious inequalities $\mu \le k \le \varepsilon$, and $\underline{x} \le x \le x$ hold, where the sense of the numbers $\mu, \varepsilon, \underline{x}$ и $\overline{x}$ is obvious. To exclude case e) from further considerations, one should request that the

$$kx \le \min_{i \in \overline{1,n}} \overline{h}_i^f .$$

holds.

Let $H = \{(k, x) \in R_+^2 : \mu \le k \le \varepsilon, \underline{x} \le x \le \overline{x}, kx \le \min_{i \in \overline{1,n}} \overline{h}_i^f \}$. The auction organizer is interested in finding such values of the parameters *k* and *x* at which the function *P*(*T*) attains its minimum on the set *H*. Since the function *P*(*T*) is a continuous function of variables *k* and *x* [6], and the set *H* is closed and bounded, this function attains its minimum on *H*.

While the problem of minimizing the function *P*(*T*) on the set *H* is quite complicated, the auction organizer is usually interested in the estimates of the function *P*(*T*) on the set *H* both from above and from below, since the knowledge of even the estimates $\underline{\omega}, \overline{\omega}$ in the inequality $\underline{\omega} \le P(T) \le \overline{\omega}$, allows one to choose appropriate values for the variables *k* and *x*, i.e., allows one to establish the maximal acceptable and a desirable (for the organizer) price for the contract being the subject of the auction.

FINDING THE UPPER AND LOWER ESTIMATES OF THE PROBABILITY *P*(*T*)

If the numbers $\underline{h}_i^f$ и $\overline{h}_i^f$, $i \in \overline{1,n}$ are known to the auction organizer or can be determined at his (her, its) request, under the assumptions regarding the functions $p(h_i)$ of the continuous variables h_i, $i \in \overline{1,n}$, the problems of finding the upper and lower estimates of the probability *P*(*T*) turn our to be relatively simple.

If *P*(*T*) takes the form (5), case d) of the mutual location of the numbers $\underline{h}_i^f, \overline{h}_i^f, kx, x$ takes place for at least one $i \in \overline{1,n}$ so that the equality

$$\min(1, \frac{\overline{h}_i^f - kx}{\overline{h}_i^f - \underline{h}_i^f}) = \frac{\overline{h}_i^f - kx}{\overline{h}_i^f - \underline{h}_i^f} \quad (6)$$

should hold when $(k,x) \in H$. If equality (6) holds for the auction participants that form a nonempty set $I \subset \overline{1,n}$, then the inequality

$$\prod_{i=1}^{n} \min(1, \frac{\overline{h}_i^f - kx}{\overline{h}_i^f - \underline{h}_i^f}) \le \min_{i \in I} \frac{\overline{h}_i^f - kx}{\overline{h}_i^f - \underline{h}_i^f}$$

holds for all $(k,x) \in H$ and, consequently, the inequality

$$\min_{(k,x)\in H} P(T) = \min_{(k,x)\in H} \prod_{i=1}^{n} \min(1, \frac{\overline{h}_i^f - kx}{\overline{h}_i^f - \underline{h}_i^f}) \le \min_{(k,x)\in H} \min_{i \in I} \frac{\overline{h}_i^f - kx}{\overline{h}_i^f - \underline{h}_i^f} \quad (7)$$

also holds.

As shown in [6], finding the value of the right hand side of the inequality (7), which coincides with $\overline{\omega}$, is reducible to the comparison of a finite number $|I|$ of real numbers.

At the same time, the obvious estimate

$$P(T) = \prod_{i=1}^{n} \min(1, \frac{\overline{h}_i^f - kx}{\overline{h}_i^f - \underline{h}_i^f}) \ge \left[\min_{i \in I} \frac{\overline{h}_i^f - kx}{\overline{h}_i^f - \underline{h}_i^f} \right]^{|I|}$$

holds so that the inequality

$$\min_{(k,x)\in H} P(T) \ge \min_{(k,x)\in H}\left[\min_{i\in I}\frac{\bar{h}_i^f - kx}{\bar{h}_i^f - \underline{h}_i^f}\right]^{|I|}.$$

also holds. From the obvious equality

$$\min_{(k,x)\in H}\left[\min_{i\in I}\frac{\bar{h}_i^f - kx}{\bar{h}_i^f - \underline{h}_i^f}\right]^{|I|} = \left[\min_{(k,x)\in H}\min_{i\in I}\frac{\bar{h}_i^f - x}{\bar{h}_i^f - \underline{h}_i^f}\right]^{|I|}$$

one can conclude that the inequality

$$\min_{(k,x)\in H} P(T) \ge \bar{\omega}^{|I|}$$

holds.

If the function *P*(*T*) takes the form (4), finding the estimates $\underline{\omega}$ and $\bar{\omega}$ for the function presents a more complicated problem. If $P(T) \neq 0$ (and this is the only case that presents interest), the equality

$$\min\left(1, \frac{\bar{h}_i^f - x}{\bar{h}_i^f - \underline{h}_i^f}\right) = \frac{\bar{h}_i^f - x}{\bar{h}_i^f - \underline{h}_i^f}$$

must hold for all *i* from a non-empty set *J* of the set $\overline{1,n}$, so that

$$P(T) = \prod_{i=1}^{n}\min\left(1, \frac{\bar{h}_i^f - kx}{\bar{h}_i^f - \underline{h}_i^f}\right) - \prod_{i\in J}\frac{\bar{h}_i^f - x}{\bar{h}_i^f - \underline{h}_i^f}$$

and, consequently, the inequality

$$P(T) \le \min_{i\in\overline{1,n}}\min\left(1, \frac{\bar{h}_i^f - kx}{\bar{h}_i^f - \underline{h}_i^f}\right) - \left[\min_{i\in J}\frac{\bar{h}_i^f - x}{\bar{h}_i^f - \underline{h}_i^f}\right]^{|J|}$$

holds. Moreover, one can be easily certain that the inequality

$$\min_{(k,x)\in H} P(T) \le \delta - \gamma ,$$

where

$$\delta = \max_{(k,x)\in H} \min_{i\in\overline{1,n}} \min(1, \frac{\bar{h}_i^f - kx}{\bar{h}_i^f - \underline{h}_i^f}), \quad \gamma = \min_{\underline{x}\le x\le \bar{x}} \left[\min_{i\in J} \frac{\bar{h}_i^f - x}{\bar{h}_i^f - \underline{h}_i^f} \right]^{|J|} .$$

also holds, and as shown in [6], finding the number γ is reducible to finding minimal values of a finite number of linear functions on a closed segment, whereas the number δ either coincides with the unit, or can be found by solving an auxiliary linear programming problem [6, 7].

Also, one should bear in mind that if the equality

$$\min(1, \frac{\bar{h}_i^f - kx}{\bar{h}_i^f - \underline{h}_i^f}) = 1 \qquad (8)$$

holds for all the auction participants, i.e., for any $i \in \overline{1,n}$, the inequality

$$\min_{(k,x)\in H} P(T) \ge 1 - \min_{(k,x)} \min_{i\in J} \frac{\bar{h}_i^f - x}{\bar{h}_i^f - \underline{h}_i^f} ,$$

holds, so finding the lower estimate of the minimal value of the probability *P*(*T*) is reducible to solving a linear programming problem on a closed segment.

If the equality (8) holds only for a subset of the set of all the auction participants, i.e., for any $i \in I \subset \overline{1,n}$, the inequality

$$\min_{(k,x)\in H} P(T) \ge \min_{x\in[\underline{x},\bar{x}]} \left[\min_{i\in J|\{i^+\}} \frac{\bar{h}_i^f - x}{\bar{h}_i^f - \underline{h}_i^f} \right]^{|J|-1} \times \min_{(k,x)\in H} \frac{x - kx}{\bar{h}_i^f - \underline{h}_i^f} ,$$

holds. In this case, finding the minimum on the segment $[\underline{x}, \bar{x}]$ is reducible to the comparison of |*J*|-1 чисел, whereas finding the minimum on the *H* is reducible to solving a linear programming problem, where i^* is a number from the set of numbers *J*, i.e., $i^* \in J \subset \overline{1,n}$.

If the boundaries $\underline{h}_i^f$ и $\overline{h}_i^f$ cannot be calculated with certainty for the auction participants to be selected to participate in the auction, the problems of finding the upper and lower estimates for the probability *P*(*T*) become more complicated. If, however, the auction organizer can indicate at least the boundaries within which the variables $\underline{h}_i^f$ и $\overline{h}_i^f$ change, for instance, $p_i \le \underline{h}_i^f \le q_i$, $\lambda_i \le \overline{h}_i^f \le \omega_i$, $\pi_i \le \overline{h}_i^f - \underline{h}_i^f \le \sigma_i$, $i \in \overline{1,n}$, $p_i, q_i, \lambda_i, \omega_i, \pi_i, \sigma_i \in R_+^1$, i.e., the vectors $\overline{h}_i^f = (\overline{h}_1^f, \overline{h}_2^f, \ldots, \overline{h}_n^f)$ and $\underline{h}_i^f = (\underline{h}_1^f, \underline{h}_2^f, \ldots, \underline{h}_n^f)$ satisfy the system of inequalities

$$P\overline{h}_i^f - Q\underline{h}_i^f \le \Theta \tag{9}$$

where *P*, *Q* are matrices and Θ is the vector вектор of corresponding dimensions, then instead of the problem $\min\limits_{(k,x)\in H} P(T)$, the auction organizer should consider the problem

$$\min_{(k,x)\in H} \max_{(\underline{h}_i^f, \overline{h}_i^f)\in\Lambda} P(T), \tag{10}$$

where Λ is a set of feasible solutions to problem (9), and find the upper and the lower estimates of the number (10). As shown in [6], the problem of finding a rigid upper estimate of the number (10) is reducible to solving an auxiliary linear programming problem for which the column generation technique can be used [7,8]. Finding a more exact estimate requires solving problems of finding the minmax of a finite number of linear-fractional functions on polyhedral sets [6]. General methods for solving such problems as those of non-smooth optimization are well known [9], whereas methods that use the specifics of these problems, for instance, finite methods, have so far been developed for the simplest cases of the problems with two linear-fractional functions [10].

The considered estimates of the minimum of both the probability *P*(*T*) and its upper and lower estimates present substantial interest for the bid organizer from the viewpoint of the correctness of choosing the contact cost *x* and the percentage of this cost (coefficient *k*) that the bid organizer can guarantee to the auction winner.

EVALUATING THE EFFECTIVENESS OF THE PROPOSED RULE FOR AUCTION ORGANIZER AND THE AUCTION PARTICIPANTS

The effectiveness of the proposed rules a)—e) depends on the relation between the chances to offer the contract that is the subject of the auction at the price *x* and at the price *kx*. It is clear that thought the bid organizer is ready to offer the price *x*, the price *kx* ($k<1$) is undoubtedly is preferable to the organizer. So if the proposed rules for determining the auction winner makes the chances of the organizer to find a contractor who would agree to take it at the price *kx* higher than those to find such a contractor at the price *x*, the rules should be considered effective from the viewpoint of the auction organizer.

Let us show that (at least under certain natural assumptions) the proposed rules do possess such a feature. To this end, let us assume that (the selected) auction participant *i* believes that all the other selected participants of the auction will submit the price for the contract that is the subject of the auction from the same segment from which this participant chooses the price to submit as his (her, its) bid.

Let

F_j be the event consisting of submitting the price by participant *j* that exceeds the price submitted by participant *i*, $i \in \overline{1,n},\ j \in \overline{1,n} \setminus \{i\}$,

T_i be the event consisting of winning the auction by participant *i* at the price that exceeds kx,

B_j be the event consisting of submitting the price that exceeds kx by participant *j*, $j \in \overline{1,n} \setminus \{i\}$,

S_i be the event consisting of winning the auction by participant *i* at the price kx, $i \in \overline{1,n}$,

$K_{I_t^\lambda}$ be the event consisting of that *t* auction participants from the set of participants $I_t^\lambda \subset \overline{1,n} \setminus \{i\}$, $|I_t^\lambda| = t$, $t \in \overline{1,n-1}$, $\lambda \in \overline{1,C_{n-1}^\lambda}$, submit the price that does not exceed kx and is the same that auction participant *i* submits, $i \in \overline{1,n}$,,

N_t be the event consisting of that participant *i* wins the auction at the price kx, provided other *t* participants submitted the price not exceeding kx,

$W_{I_t^\lambda}$ be the event consisting of that *t* auction participants from the set $I_t^\lambda \subset \overline{1,n} \setminus \{i\}$, $|I_t^\lambda| = t$, $t \in \overline{1,n-1}$, $\lambda \in \overline{1,C_{n-1}^\lambda}$ submit the price that coincides with the price submitted by participant *i* and exceeds *kx*, whereas all the other participants submit prices that exceed *kx*,

H_t be the event consisting of that participant *i* wins the auction, provided that other *t* participants from the set $I_t^\lambda \subset \overline{1,n} \setminus \{i\}$, $|I_t^\lambda| = t$, $t \in \overline{1,n-1}$, $\lambda \in \overline{1,C_{n-1}^\lambda}$

submitted one and the same price that exceeds *kx* and coincides with the price submitted by auction participant *i*, and all the other participants submit prices that exceed *kx*.

Then the events T_i and S_i can be presented as follows:

$$T_i = \prod_{j=1,\, j\neq i}^{n} (B_j F_j) + \sum_{t=1}^{n-1} (\sum_{\lambda=1}^{C_{n-1}^{t}} W_{I_t^\lambda} \prod_{j\in J_t^\lambda} B_j F_j) H_t \,,$$

whereas

$$S_i = \prod_{j=1,\, j\neq i}^{n} B_j + \sum_{t=1}^{n-1} (\sum_{\lambda=1}^{C_{n-1}^{t}} K_{I_t^\lambda} \prod_{j\in J_t^\lambda} (B_j + G_j)) N_t + \prod_{j=1,\, j\neq i}^{n} G_j \,,$$

where $|I_t^\lambda| = t$, $|J_t^\lambda| = n-1-t$, $I_t^\lambda \cup J_t^\lambda = \overline{1,n} \setminus \{i\}$,

$I_t^\lambda \cap J_t^\lambda = \varnothing,\ \lambda \in \overline{1, C_{n-1}^{\lambda}}$.

Under the assumption regarding the continuity of the random variables h_i, the inequalities

$$P(T_i) = P(\prod_{j=1,\, j\neq i}^{n} B_j) P((\prod_{j=1,\, j\neq i}^{n} F_j) / (\prod_{j=1,\, j\neq i}^{n} B_j)) < P(\prod_{j=1,\, j\neq i}^{n} B_j),$$

and

$$P(S_i) = P(\prod_{j=1,\, j\neq i}^{n} B_j) + P(\sum_{t=1}^{n-1} (\sum_{\lambda=1}^{C_{n-1}^{t}} K_{I_t^\lambda} \prod_{j\in J_t^\lambda} (B_j + G_j)) N_t) +$$

$$P(\prod_{j=1,\, j\neq i}^{n} G_j) > P(\prod_{j=1,\, j\neq i}^{n} B_j)$$

hold so that the inequalities

$$P(T_i) < P(\prod_{j=1,\, j\neq i}^{n} B_j) < P(S_i), \tag{11}$$

also hold. The latter inequality means that the chances of any auction participant to win the auction at the price exceeding *kx* are smaller than those to win the auction at the price *kx*. Thus, the proposed rules serve the interests of the auction organizer in the sense that lets him (her, it) hope that the contract that is the subject of the auction will be taken at the price that is smaller than the maximal one (*x*) at which organizer can afford to pay for the contract.

It turns out that the proposed rules serve the interests of the auction participants (in a certain sense) as well. Let us compare the chances of every auction participant to win the auction at the price *kx*, which is desirable for the auction organizer, under the proposed rules and under the traditional rule, when the bid with the lowest price wins.

Let

D_i be the event consisting of that auction participant i wins the auction at the price *kx* under the above-mentioned traditional rule;

Q_t be the event consisting of that t auction participants submit the price *kx*, $I_t^\lambda \subset \overline{1,n} \setminus \{i\}$, $|I_t^\lambda| = t$, $t \in \overline{1, n-1}$, $\lambda \in \overline{1, C_{n-1}^{\lambda}}$;

M_t be the event consisting of that auction participant i wins the auction at the price *kx*, assuming that other *t* auction participants also submitted price kx.

The event D_i can be presented as follows:

$$D_i = \prod_{\substack{j=1 \\ j \neq i}}^{n} B_j^2 + \sum_{i=1}^{n-1} (\sum_{\lambda=1}^{C_{n-1}^t} Q_{I_t^\lambda} \prod_{j \in J_t^\lambda} B_j F_j) M_t$$

so that the equality

$$P(D_i) = P(\sum_{\substack{j=1 \\ j \neq i}}^{n} B_j)$$

and the inequality

$$P(T_i) < P(D_i) < P(S_i). \quad (12)$$

holds.

The latter inequality means that for each auction participant, the chances of winning the auction at the price *kx* under the proposed rules are higher than those under the traditional rule. Thus, the proposed rules are more attractive for the potential auction participants than the traditional rule.

Thus, the proposed rules are advantageous for both auction participants from the first group and the organizer of the auction.

The proposed economic mechanism serves two particular goals of the auction organizer a) not to discourage reputable potential participants from submitting their bids that reflect their values of the contract and their ability to implement it with the required quality at the submitted price, and b) to encourage all the potential participants to study both the market and their competitors in submitting the bids rather than submitting the bids that reflect only how they value the contract.

It turns out that a slight modification of the proposed rules can make it even less vulnerable to potential corrupt activities. That is, if *x* is the maximal price, the winning price is guaranteed to be within the segment [*kx, x*], where $k < 1$, and the winner is determined as follows:

f) if all the submitted prices do not exceed *kx*, and not all of them are the same, then the participant who has submitted (or a participant from among those who have submitted) the price that is either next smaller than the closest to *kx* price (if the price *kx* has not been submitted by the participants) or next smaller than *kx* (if the price *kx* has been submitted by at least one of the participants) is declared the winner, and the winning price is *kx,*

g) if all the submitted prices are not lower than *kx*, and not all of them are the same, then the participant who has submitted (or a participant from among those who have submitted) the price that is either next greater than the closest to *kx* price (if the price *kx* has not been submitted by the participants) or next greater than *kx* (if the price *kx* has been submitted by at least one of the participants) is declared the winner, and the winning price is the price submitted by the winner,

h) if at least one from among all the submitted prices is smaller than *kx*, whereas at least one from among the other submitted prices is greater than *kx*, the winner is either determined by rule f) from among the participants who have submitted the prices not exceeding *kx* (if at least two different prices not exceeding *kx* have been submitted) or the participant who has submitted (or a participant from among those who have submitted) the price that is smaller than *kx* is declared the winner (if only one price smaller than *kx* has been submitted), and the winning price is *kx* in both cases,

i) if several participants submitted the same winning price, the winner is determined by an additional procedure,

j) if all the submitted prices exceed *x*, the sealed-bid auction is considered failed.

It turns out that under the same natural assumptions, inequalities (11) and (12) hold for both rules f)—j). Also these rules keep the submission of dumping prices by the auction participants economically unprofitable. However, rules f)—j) make unreliable any guarantees to win the auction that the auctioneer can give to any individual auction participant and reduce the chances of forming a corrupt pair “the auctioneer—an individual auction participant”.

The proposed rules reduce the chances of establishing corrupt relationships between the auctioneer and one individual one-step, sealed-bid auction participant. Nevertheless, they cannot block (or reduce the chances of) forming corrupt relationships among the auctioneer and more than one individual participant or with a cartel acting as a collective auction participant. One, however, should bear in mind that establishing corrupt relationships with a group of individuals is always more risky than in the case with only one individual auction participant. Finally, one should bear in mind that calculating a maximal acceptable price for the contract can be viewed as finding the starting price that sometimes must be announced to hold the auction procedure [7].

CONCLUDING REMARKS

1. The proposed approach to reducing the risks associated with allocating and implementing procurement contracts allows the administration responsible for spending taxpayers' money in procuring goods and services to choose the best available strategy when a) it has financial resources to explore whether the chances of certain unfavorable events to occur can be reduced, and b) when it does not have them. If one considers a strategy a combination of the type of a competition procedure and the type of a contract that it may choose to offer to the auction winner, the chances of successfully implementing the contract can be estimated (for instance, by the experts) with the use of simple probabilistic considerations. When the administration can afford to spend some amount of financial resources for the exploration of the risks associated with the use of a set of particular strategies, the best allocation of these financial resources can be found by solving a geometric programming problem. Methods of geometric programming have proven to be more effective in solving

mathematical programming problems with functions-posynomials, which are present in problem (2), than general non-linear programming methods.

2. The proposed rules for choosing the winner in a one-step sealed-bid auction make economically unprofitable the submission of dumping prices for the contract that is the subject of a sealed-bid auction. Also, these rules reduce the chances of the appearance of corrupt relationships between the auctioneer, acting on behalf of the government in the public procurement goods and services, and an individual participant of a sealed-bid auction.

REFERENCES

1. A. Belenky (2001). An Approach to Planning an Advertising Campaign of Goods and Services, *Computers and Mathematics with Applications*, 67 (2): 993-1008.

2. A. Belenky (2005). Competitive Strategies of U.S. Presidential Candidates in Election Campaigns, *Computers and Mathematics with Applications*, 49 (1): 53-71.

3. G. Piga (ed.) (2007).The Economics of Public Procurement, Palgrave McMillan, 1st edition, Houndmills, Basingstoke, Hampshire, UK

4. V. Nagali, et. al (2008). Procurement Risk Management (PRM) at Hewlett- Packard Company, Interfaces (38) N1: 51-60.

5. R. Duffin, E. Pterson, C. Zenner (1967). Geometric Programming. John Wiley & Sons, New York.

6. A. Belenky (2006). Two Rules of a Sealed Ceiling Bid and Their Analysis by Mathematical Programming Techniques, *Computers and Mathematics with Applications* 52 (2):1711-1732.

7. A. Belenky (2009). A Rule for Determining the Winner in Sealed Ceiling Bids and Its Mathematical Properties, *Economic Journal of Higher School of Economics*, 13 (1): 3-18 (in Russian)

8. L. Lasdon (1970) Optimization Theory for Large Systems, McMillan, New York.

9. V. Demyanov, V. Malozemov (1990). Introduction to Minimax, Dover Publications, New York.

10. A. Belenky (1980). Search for Mini-max of Two Monotone Functions of a Polyhedral Set. *Automation and Remote Control*, 43 (11): 1389-1393.