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A SIMPLE MODEL OF FRAMEWORK AGREEMENTS: COMPETITION AND EFFICIENCY[1]

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ABSTRACT. Directive 2004/18/EC provides a uniform and harmonized legal framework for conducting public procurement in Europe. In this paper we deal with one of the new institutes introduced by the Directive, namely the framework agreements (FAs). We set up a two-stage model in which a central purchasing body (CPB) first concludes an *incomplete* FA with at least three firms. Competition is then reopened by one among several contracting authorities (CAs). We find that admitting a higher number of firms is efficiency enhancing, since more final users are likely be served. Nevertheless, a higher number of admitted firms induce less aggressive competition at the first stage, leading to higher prices (lower savings). We provide numerical solutions to the trade-off between savings and efficiency.

INTRODUCTION

The Directive 2004/18/EC of the European Parliament and of the Council of March 31, 2004 "On the Coordination of Procedures for the Award of Public Works Contracts, Public Supply Contracts and Public Service Contracts" provides a unified legal framework for conduction of public purchases of goods and services in Europe. It has been (or it is currently being) transposed and applied in all EU Countries.

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Beyond reorganizing public procurement through a uniformed and harmonized legislation, the Directive also aims at providing European governments, contracting authorities and central purchasing bodies with more flexible and dynamic procurement tools. Two main goals are admittedly pursued. First, by heavily relying on the use of information technologies they aim at reducing the length (and, plausibly, the cost) of procurement processes. Second, they aim at improving efficiency and flexibility in order to better fit the complex and continuously changing needs of government. In particular, the spirit of innovative institutes such as framework agreement, dynamic purchasing system, competitive dialogue and electronic auction is that of a dynamic (multi-stage in some cases) process in which either contract clauses (as in the case of framework agreements) or aspects of the economic offer (electronic auction) can be modified at different stages before each procedure is concluded.

In this article, we present a stylized model of framework agreements. The Directive defines a framework agreement (FA) as "an agreement between one or more contracting authorities and one or more economic operators, the purpose of which is to establish the terms governing contracts to be awarded during a given period, in particular with regard to price and, where appropriate, the quantity envisaged."

One of the most innovative features of such an institute is that the Directive allows the contracting authorities/purchasing bodies to conclude FAs even with more than one economic operator.[2] Furthermore, it also allows all the terms of the contracts to be awarded to be not laid down in the FA. In what follow, we will refer to this case as to an "incomplete FA".

Where this is the case, and where the FA is concluded with several economic operators, the contracting authority going to award a specific contract based on the FA shall reopen the competition "on the basis of the same and, if necessary, more precisely formulated terms, and, where appropriate, other terms referred to in the specifications of the framework agreement." Thus, the contracting authority shall invite the operators capable of performing the contract to submit tenders for a specific contract (call-off). Finally, the contract shall be awarded "to the tenderer who has submitted the best tender on the basis of the award criteria set out in the specifications of the framework agreement."

Such a flexible procedure seems a suitable purchasing process for a set of contracting authorities whose preferences are somewhat heterogeneous at any specific point in time and/or are likely to change over time. It is worthwhile noticing that CAs, say schools buying chairs for kids, may differ not only with respect to some aspects of the supply contract that are *subjective* (that is, school “A” prefers red chairs, whereas school “B” prefers blue chairs), but also with respect to some other *objective* dimensions, such as the distance between the school and the supplier’s warehouse where chairs are stocked. It is exactly this *objective* source of heterogeneity among CAs that inspires our simple model. So let us continue with the example of schools buying chairs. Suppose that schools are scattered over a particular geographical region: some of them are located in easily accessible towns whereas other schools serve mountain villages. All schools agree on the same technical characteristics of chairs they plan to buy over a certain period of time, although their purchase decisions are not necessarily synchronized. Thus, from suppliers’ perspective the most relevant source of uncertainty is how much each single school will be buying. This dimension matters since it has a direct impact on transportation costs. If a particular firm is located in the neighborhoods of an urban area its transportation costs will be lower when serving a school down-town rather than one up on a rocky mountain.

How would one conceive a FA in such circumstances? All interested schools may conclude a FA with a certain number of suppliers that are selected on the basis of the *maximum price* for chairs. The FA does not specify precisely delivery conditions, which will differ from one call-off to another. At a later stage, each single school will reopen the competition among the set of admitted firms. The latter will be then in a position to target their tender since they know with certainty the delivery conditions, which in turn implies they can compute precisely transportation costs. The question then becomes: for a given number of active firms in the market, with how many of them should the schools conclude a FA?

In what follows, we will show that a clear trade-off between *competition* (at the entry stage) and *efficiency* arises in such a two-stage process, where the relevant dimension of efficiency is the likelihood at which schools are served. When the FA is concluded with a low number of firms (the Directive prescribes this number to be at least 3) competition among all competitors is likely to be tough.

However, when competition is reopened at a later stage it may happen that none of the admitted firms is interested in a specific contract, thus raising the risk that a contracting authority is not served. On the other hand, if a FA is concluded with a large number of firms – where large is relative to the set of all active firms in the relevant market – competition at the entry stage becomes softer, but the risk that no firm is interested in any subsequent call-off is, in general, significantly reduced.

To the best of our knowledge this is the first attempt to formally analyze some of the economic forces at play within the legal institute of framework agreements. Our approach, however, bears some similarities with Ye (2007). That author studies a model of two-stage auction with incomplete information where bidders can learn at some cost additional information about the value of the asset at sale after the first stage. The auctioneer may then want to limit entry to avoid too many bidders to invest in acquiring information. In our model, instead, the central purchasing body may want to restrict entry in order to balance the trade-off between efficiency (that is, the likelihood that each CA is served) and competition.

In this article, first we set the primitives of the model, illustrate how the main economic forces work by using two examples and characterize the equilibrium of the general model. Second, we conduct a welfare analysis in order to measure the trade-off explained above. Finally, we elaborate on the extension of the model with entry costs.

THE MODEL

Let us consider a two-stage game in which potential suppliers (*N*) compete to serve supply contracts to contracting authorities (*M*). Firms are located on the interval [0,1], and are equidistant from one another and from the extreme points, 0 and 1, of the interval. CAs lie on the same segment, and are also equidistant from one another. However, unlike firms, two of them are also located on each extreme point of the line. Formally, the position of the *i*-th firm is given by

$$x_i = i\frac{1}{(N+1)}, \qquad i=1, \ldots, N$$

while the position of the *j*-th CA is

$$y_j = (j-1)\frac{1}{(M-1)}, \qquad j=1, ..., M.$$

For each configuration of the economy, described by a couple of numbers (*N*, *M*), it may be useful to introduce the following distance:

$$\Delta(M,N) = \frac{1}{(N+1)(M-1)}$$

that allows us to rewrite the locations of both firms and CA in a convenient way:

$$x_i = i(M-1)\Delta(M,N), \qquad i=1, ..., N$$

$$y_j = (j-1)(N+1)\Delta(M,N) \qquad j=1, ..., M.$$

Consistently with the spirit of Directive 2004/18/CE, we consider a Framework Agreement concluded by a central purchasing body (CPB) with *n* firms. Final demand arises only once from one of *M* CAs with probability *1/M*. We assume firm *i*'s cost to supply the *j*-th CA to be given by its transportation cost, which is equal to the distance between the firm and the CA. Thus the supply cost can be written as $c_{i,j}=|x_i-y_j|$. Moreover, suppose all the CAs to have the same evaluation *V* of the good/service, so that the utility for a CA from purchasing the good is *u=V-p*, where *p* is the price paid for the contract.

As customary in models of horizontal differentiation, there are two conceivable ways of interpreting the distance between each single firm and a CA. The first interpretation – that we explained in the Introduction section above – is a pure transportation cost; the second interpretation – perhaps more consistent with Hotelling's original formulation – represents the distance in the space of each CA's preferences, namely how far away is any firm's product from a specific CA's "ideal" product.

Sellers have complete information: they know the location of both the other firms and the CAs, their evaluation *V* of the good/service and their own cost structure. They also know that only one of the *M* CAs will demand the good and award the contract, with probability *1/M*. This information structure is also known to the CPB. Before moving to the timing of the game, it is worth spending a few comments on the information structure. The assumption of complete information among firms captures a mature market where competitors know each other reasonably well. Basically, firms share the same technology to produce their final product (think of the chairs in our introductory example), but they differ with respect to their transportation costs which depend on the distance between each school and their warehouses. Thus our assumption of complete information reduces to each firm knowing the location of any other competitor's warehouse, which sounds a reasonable assumption in the case of "mature" markets. The assumption of complete information on the part of the CPB is merely instrumental to conduct a welfare analysis.

The timing and the rules of the game are as follows:

$t=1$

The CPB fixes the number $n \leq N$ of winners of the FA, and makes it public. Each seller i submits her sealed-bid offer b_i on the price of the contract, without knowing yet which one of the CAs will make a call-off. The FA is concluded with the n sellers who submit the n lowest prices. In the case of equal offers, a tie-breaking rule selects the winners at random with equal probability. Let I denote the subset of firms i with whom the FA has been concluded.

At the end of the first stage both the names of the winners and their bids are made public.

$t=2$

After the FA has been concluded, only one CA $j' \in \{1, 2, \ldots, M\}$ will make a call-off. The administration j' will invite the n winners to compete again in a sealed-bid competitive tendering. For each firm $i \in I$, the price b_i submitted at the first stage represents an upper bound for the second stage bid B_i. Thus, whenever a firm among the selected ones decides to compete in the second stage she can only do that by submitting a price $B_i \leq b_i$. However, a firm $i \in I$ can choose as well not to make any offer. In this case, we denote $B_i = \varnothing$.

The call-off is awarded to the firm which makes the lowest bid. Ties are broken fairly by a random device. We can now formalize the notion of strategy.

Definition 1 (Strategy). For a generic firm *i*, with *i=1, .., N*, a strategy in the two stages game is a profile of actions $s_i=(b_i, B_i(b, I, j')) \in [0, +\infty] \times ([0, bi] \cup \{\varnothing\})$, where:

- $b_i \geq 0$ is the price submitted at the first stage;
- $B_i \in [0, b_i] \cup \{\varnothing\}$ is the price submitted at the second stage ($\varnothing$ if firm *i* refrains from bidding);
- $b=(b_1,..., b_n)$ is the vector of first stage bids of all the firms;
- *I*={*i*/ the firm *i* has concluded the FA}; and
- $j' \in \{1, ..., M\}$ is the CA who makes the call-off at the second stage.

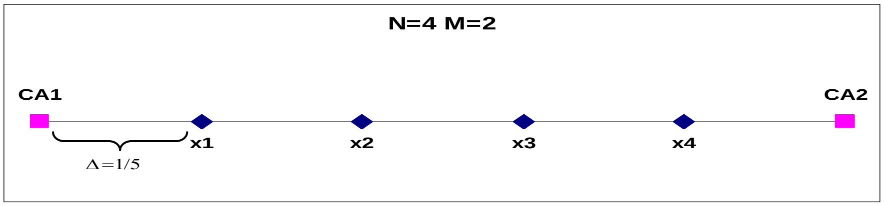
The above definition is standard in defining a strategy as a complete plan of actions contingent to the history of the game leading to each node. In our case, as soon as the second stage is reached, a (non-terminal) history of the game is fully identified by the vector of first stage bids *b*, the set *I* of firms qualified for the second stage,[3] and the CA *j'* awarding the specific contract.

In order to ease exposition and reading we will focus our attention only on the plans of actions effectively played at equilibrium. In particular, rather than characterizing each firm's complete equilibrium strategy we will describe those actions played along the equilibrium path. The equilibrium concept we will use is that of subgame perfect equilibrium (SPE).

A convenient, additional assumption is to fix ε as the smallest monetary unit available for the bids. We also assume ε to be arbitrarily small (so that $\varepsilon << \Delta$) and $\Delta = k\varepsilon$, where *k* is an integer number.[4] This assumption is often implicit in dealing with games where money is assumed to be a discrete variable, so that the term "ε" is usually taken away from the notation. Nevertheless, we will keep it in our notation, in order to make the identification of the winning bids easier.

Before solving the game for a generic configuration (*N,M*), we find it useful to discuss two specific examples that will make clear the emergence of the most meaningful economic trade-off.

Example 1: *N=4, M=2*



In this configuration, two CAs are located at the extremes of the segment, and $\Delta(N,M)=\Delta(4,2)=1/5$ simply coincides with the distance between two neighboring firms. Firms 1 and 4 are the most efficient in supplying, respectively, CA 1 and 2.

We first consider the case with $n=3$, where the FA is concluded with three firms so that one firm is left out from the second stage. Assume that at the second stage CA 1 invites the three winners to a competitive tendering to purchase the good/service. The lowest bids that firms 2 and 3 may submit to cover transportation costs are $B_2=2\Delta$ and $B_3=3\Delta$, respectively (provided that $2\Delta\leq b_2$ and $3\Delta\leq b_3$). Thus, if firm 1 is a winner of the FA, its optimal second stage bid is $B_1=\min\{3\Delta, b_1\}$ if firm 2 has been left out, or $B_1=\min\{2\Delta, b_1\}$ otherwise. Notice that, provided that $b_1>\Delta$, in both cases firm 1 gets a positive profit equal to $b_1-\Delta$. Observe that this outcome is analogous to an asymmetric Bertrand competition. The only difference may arise when the constraint $B_i\leq b_i$ is binding.

Since the same reasoning applies for firm 4 when the call-off is made by CA 2, we conclude that, if both firms 1 and 4 qualify for the second stage, firms 2 and 3 will never get the specific contract, while firm 1 and 4 will always supply, respectively, CA 1 and 2. Hence, it is immediate that, at equilibrium, firms will submit the following first-stage bids: Firms 1 and 4 will offer $b_1=b_4=2\Delta-\varepsilon$. Indeed this is their maximum offer able to guarantee qualification for the second stage, by undercutting the other two competitors. Firms 2 and 3 will submit $b_2=b_3=2\Delta$. Observe that, for $i=2, 3$ no bid $b_i>2\Delta$ could be part of a first-stage equilibrium strategy, as it would make a deviation for players 1 and 4 profitable. Indeed, in this case, the latter could increase their bid in order to increase their profit by relaxing their constraint $B_i\leq b_i$. Therefore, firms 1 and 4 win the FA with probability 1, and the third winner is randomly selected (with probability 1/2) between firms 2 and 3.

At the second stage, if the call-off is made by CA 1, firms 3 and 4 will not make any offer (since the constraint $B_i \leq b_i$ would prevent them from covering the transportation cost) while firm 1 and 2 confirm their first stage price, so that $B_1=b_1=2\Delta-\varepsilon$ and $B_2=b_2=2\Delta$.[5] Thus firm 1 wins the specific contract, supplies CA 1 and gets a strictly positive profit equal to $\Delta-\varepsilon$. This profit clearly arises because firm 1 has a competitive advantage in serving the CA 2. Analogously, in the case where the contract is awarded by CA 2, all the results will replicate with firm 4, rather than firm 1, serving the contract.

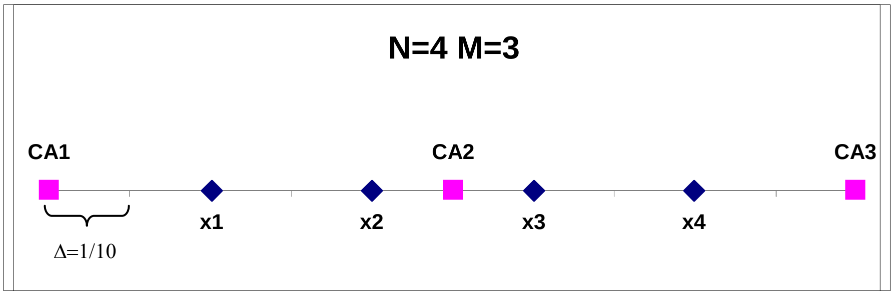
To sum up, the structure of the game is quite close to two "embedded" asymmetric Bertrand games in which firm 1 has a cost advantage with respect to firm 2, and firm 4 has a cost advantage with respect to firm 3. At the first stage, firms do not know which game will become relevant since demand is unknown. However, both firm 1 and firm 4 will make a first-stage bid so as to cut-off their closest competitor.

Finally notice that there exist other SPE. For instance, one can easily verify that the following strategy profile constitutes an equilibrium as well: $b_1=b_4=\Delta$, $b_2=b_3=0$; $\forall j' \in \{1,2\}$ $B_1=B_2=B_3=B_4=\emptyset$. Such an equilibrium hinges on firms using weakly dominated strategies. In what follows we will rule out such economically meaningless equilibria by assuming that firms play undominated strategies.

We now turn to the case $n=4$, where the FA is concluded with all the four firms. In this case, the first stage matters even less than in the previous example.[6] Once the CA exerting the demand is revealed, the game results again in an asymmetric Bertrand competition, whose final outcome is the same as in the case $n=3$: when one of the CAs makes a call-off, its closest firm (either firm 1 or firm 4) wins the specific contract at price $B=2\Delta-\varepsilon$.

Before moving to the second example, it is worth making some comments about welfare. Regardless of whether $n=3$ or $n=4$, as soon as competition is reopened both CAs are served at a price $B=2\Delta-\varepsilon$.

Example 2: *N=4, M=3*



In this case, $\Delta=\Delta(4,3)=1/10$. A third CA is now located in the middle of the line, firms 2 and 3 being the closest to the central CA.

We first consider the case $n=3$. If, at the second stage, CA 2 makes a call-off, both firm 2 and 3, regardless of their first-stage bid and provided they have won the FA, engage in a (symmetric) Bertrand competition whose final outcome is $B_2=B_3=\Delta$. The winner of the contract is selected randomly among them, and makes no profit. On the other hand, the minimum cost for firms 1 and 4 to supply their closest CA is 2Δ, so that firms 2 and 3 can undercut their competitors in stage 1 by simply bidding 2Δ and can be admitted to the second stage for sure. Hence, either firm 1 or 4 will be excluded. In addition, firms 1 and 4 would get a positive profit if they could serve their closest CA at a price $B>2\Delta$. This triggers aggressive competition to enter the second stage, leading both firm 1 and 4 to bid the minimum cost 2Δ.

Therefore it is easy to see that the following strategy profile constitutes a SPE of the game: $b_1=b_4=2\Delta$, $b_2=b_3=2\Delta-\varepsilon$, $B_2=B_3=\Delta$, $B_j=2\Delta$, where $j\in\{2,4\}$ depending on whether firm 2 or firm 4 is randomly selected at the first stage. Notice that, unlike the previous example, no seller makes strictly positive profit at equilibrium. This comes from the symmetry of the configuration: firms 2 and 3, being equidistant from CA 2, make no profit as a consequence of a symmetric Bertrand competition, while firm 1 and 4 erode their potential profit in order to be admitted to the second stage.

Thus, on average, purchasing prices for CAs are lower than in Example 1. Nevertheless, only CA 2 is now served with probability 1, whereas CA 1 and CA 3 are served with probability 1/2, which is the likelihood that firms 1 or 4 are randomly selected at the end of stage 1.

Unlike the previous example, here the number *n* of firms included in the FA *does* matter. Consider the case *n=4*. Again, if all the firms enter the second stage, there is no competition at the first stage whereas Bertrand competition occurs (either symmetric or asymmetric) to win the specific contract. As to the case *n=3*, nothing changes when CA 2 makes a call-off. On the contrary, when the call-off is made by CA 1 (respect. CA 3), firm 1 (respect. firm 4) can exploit the competitive advantage over its closest competitor. In these cases the winning bid is $B_1=4\Delta-\varepsilon$ if demand arises at CA 1 and $B_4=4\Delta-\varepsilon$ if demand arises at CA 2, yielding a net profit equal to $2\Delta-\varepsilon$. The purchasing price for CA 1 and CA 3 is higher than in the case *n=3* where, because of the competition at the first stage, B_1 (or B_4)$=2\Delta=b_1$ (or b_4). However, if *n*=4 all CAs are served with probability one. Hence a trade-off emerges between the cost and the probability of being served.

Assume, for instance, that CA 1 makes a call-off. Given that its evaluation of being served is *V*, its expected payoff (neglecting ε) is:

$1/2(V-2\Delta)$ if $n=3$

$V-4\Delta$ if $n=4$

where the factor 1/2 takes into account the probability of CA 1 being supplied (equal to the probability that firm 1 wins the tie break with firm 4 at the first stage). The payoff in the case *n=4* is higher if $V\geq 6\Delta$. The interpretation is that the higher the value *V* of the supply for the CA, the more profitable to admit more firms to the second stage, despite a lower first stage competition may induce higher supply costs. We will come back to this point later on, where we analyze this trade-off in the general setting *(N, M)*, from the point of view of the CPB.

Finally, also in this case, multiplicity of equilibria arises. Furthermore, we can even find some equilibria with no bids below the cost. For instance, it is easy to check that, with *n=3*, any firm 2's and 3's first-stage bids in the interval $[\Delta, 2\Delta-\varepsilon]$ can yield to a SPE. However, this class of equilibria leads to the same final outcome of the game in terms of both supply price and probability of supplying each CA.

The Equilibrium in the General Case

We now turn to the general case. Our goal is to describe the equilibrium of the model for each configuration *(N, M)* and for each $3 \leq n \leq N$.

We first need to introduce some additional piece of notation. For each firm *i*, let d_i denote the distance from its closest CA. Thus for firm *i*, d_i also represents the cost of serving that CA. Let d^n be the minimum distance such that the number of firms with $d_i \leq d^n$ is greater than or equal to *n*. In other words, this means that at least *n* firms are less far from (at least) one CA than d^n and that no $d<d^n$ exists such that at least *n* firms are closer than *d* to one CA. Finally, d^{n+1} is defined as the lowest d_i greater than d^n.

In order to rule out economically meaningless equilibria (like the ones mentioned in Example 1) we assume that firms play undominated strategies, namely that at the first stage they never submit bids that are below their lowest transportation cost. As briefly discussed in example 2, multiplicity of equilibria might arise as well. Such equilibria are outcome equivalent and differ only with respect to the bid submitted at the first stage. So in Proposition 1 below we characterize the SPE which involves the highest bid at the first stage. We are now ready to state the following preliminary result.

Lemma 1

When competition is reopened within a framework agreement, an admitted firm will only respond to a call-off made by its closest contracting authority.

Proof

We have to prove that any firm entering stage 2 of the game will be able to serve at most one CA, namely the closest one. First, observe that any firm reaching stage 2 will realize a non-negative payoff only if it is able to serve at least one CA. A necessary condition for this is $b_i \geq d_i$..

We have to show that any firm admitted to the second stage cannot serve both its closest and second-closest CA. Notice that the distance between two neighboring CAs is $(N+1)\Delta$. From the definition of d_i, the distance between the *i*-th firm and its second-closest CA is

$(N+1)\Delta-d_i \geq (N+1)\Delta/2 \geq d_i$. We now show that no firm can be admitted to the second stage with a bid higher than $(N+1)\Delta/2$.

We need to consider two cases:

1. Suppose that $b_k > (N+1)\Delta/2$ were to allow firm k to enter stage 2 with probability 1. Then there must be $(N-n)$ firms whose (first-stage) bids are strictly higher than b_k. However, any firm i belonging to the set of excluded firms would have a profitable deviation by submitting a first-stage bid marginally below b_k. Such a bid would ensure a positive expected payoff since $b_i > (N+1)\Delta/2 > d_i$.
2. Consider the case where $b_k > (N+1)\Delta/2$ makes firm k tie with other T firms at stage 1 and L firms, $L < n$, bid below b_k. Consequently firm k is selected for the second stage with probability $1/T$. However, firm k has an incentive to deviate by submitting an offer slightly below b_k, since a decrease of the potential profit due to a slightly lower bid is more than compensated by the increase (from $1/T$ up to 1) of the probability of being selected.

Therefore any firm i selected at the first stage submits an offer b_i such that $b_i \leq (N+1)\Delta/2 \leq (N+1)\Delta-d_i$ and thus, since it must be $B_i \leq b_i$, is able to serve only its closest CA when competition is reopened (Q.E.D.).

Lemma 1 is instrumental to the characterization of the equilibrium strategies in the two-stage game. It basically tells us that each firm “targets” only one CA. Consequently, the set of firms admitted to the second stage includes those closest to *any* CA. In order to maximize the expected profit, at the first stage the same firms submit exactly the bid that cuts off $N-n$ competitors. This is formally shown in the following.

Proposition 1

The following plans of actions are part of a SPE of the two-stage game:

Stage 1

1.a) If the number of firms such that $d_i \leq d^n$ is equal to n, then the first stage bids are:

$b_i=d^{n+1}-\varepsilon$ for all i such that $d_i \leq d^n$

$b_i=d_i$ for all i such that $d_i>d^n$

1.b) If the number of firms such that $d_i \leq d^n$ is greater than n, then the first stage bids are:

$b_i=d^n-\varepsilon$, if $d_i<d^n$

$b_i=d_i$, otherwise.

Stage 2

Denote $I=\{i$ / the firm i has entered stage 2$\}$. For each firm $i \in I$ let $j(i)$ be its closest CA. Firm i's equilibrium bid writes:

2.a) $B_i=\varnothing$, if demand arises from any CA other than $j(i)$;

2.b) $B_i=b_i$, if demand arises from $j(i)$ and $\forall\ i' \in I$ such that $i' \neq i$, $|x_{i'}-y_{j(i)}| \geq b_i$;

2.c) $B_i= max\ (d_i, min_{i' \neq i}\ (|x_{i'}-y_{j(i)}|)-\varepsilon)$, otherwise.

Proof

We first show that 2.a, 2.b, and 2.c describe equilibrium bids at the second stage of any firm with whom a FA is concluded.

2.a) The optimality of the equilibrium bid follows from the Lemma.

2.b) Assume that CA $j(i)$ reopens the competition. If no firm $i' \neq i$ – also admitted to the second stage – is located at distance from $y_{j(i)}$ lower than b_i, firm i has no reason for reducing her first-stage bid, so that she can maximize her surplus $B_i-|x_i-y_{j(i)}|= B_i-d_i$ without any risk of losing the competition.

2.c) If the first-stage constraint puts two (or more) firms in the condition to serve the contract, the optimal strategy for firm i is to bid slightly below its most efficient competitor's cost– namely $min_{i' \neq i}(|x_{i'}-y_{j(i)}|)$ – in the case that i is the closest firm to the CA $j(i)$. If this is not the case, the best firm i can do is to bid her own cost d_i.

We now roll back to the first stage.

Case 1.a

Assume that each one of the *(N-n)* less efficient competitors, i.e. firms with $d_i>d^n$, bid $b_i=d_i$. From the definition of d^{n+1}, the most efficient among the latter will be such that $d_i=d^{n+1}$. This implies that, for the *n* most efficient firms, the best response is to offer $b_i=d^{n+1}-\varepsilon$. In fact, in this way they guarantee themselves to enter stage two with certainty while making their second stage constraint $B_i \leq b_i$ as relaxed as possible. This will allow them to maximize their expected profit.

We have now to prove that, given this strategy of the *n* most efficient firms, the other (*N-n)* competitors have no incentive to deviate from offering $b_i=d_i$. By submitting such a bid the less efficient firms are excluded from the FA and get zero profit. By lowering their bid, they would not raise their profit. Indeed they would enter the second stage, but they would be unable to cover the supply cost to serve any CA. This means that bidding below d_i is a dominated strategy. Finally, notice that they are indifferent between any bid b_i such that $b_i \geq d_i$, but a bid higher than d_i cannot be part of an equilibrium. If this was the case, it would become profitable for each one of the *n* most efficient firms to increase her own bid in order to make its second stage constraint less stringent. Thus we conclude that strategies in 1.a) are equilibrium strategies.

Case 1.b

Here the number of firms whose d_i is lower than d^n is greater than *n*. This implies that we have to consider three groups of firms. Assume that there are *H* firms (with $H<N-n$) whose d_i is greater than d^n; *L* firms (with $L<n$) whose d_i is lower than d^n; $T=N-(L+H)$ firms whose d_i is equal to d^n. Now assume that both the *T* firms and the *H* firms submit a price equal to their own d_i at the first stage.

As in the previous case, the *L* most efficient firms' best reply is to slightly undercut their competitors in order to enter the second stage with probability 1 while maximizing their expected payoff. This is achieved through an offer equal to $d^n-\varepsilon$. For a firm in this group no profitable deviation exists. Lowering its bid would reduce expected profit. On the other hand, an higher bid would exclude a firm from the second stage in the case $b_i>d^n$, while in the case $b_i=d^n$ her likelihood to be selected would shift from *1* to *1/(T+1)* (as in this case she would tie with the other *T* firms). The increase of the expected profit due to the increase by ε of the bid is more then compensated by the decrease due to lower probability of being selected, in force of our assumptions on ε.

Notice now that both the T tying firms and the H less efficient firms, by bidding their d_i as assumed above, get zero profit. It is easy to realize that they have no incentive to deviate, as the same argument pointed out about the N-n least efficient firms of the case 1.a) applies. Thus the plans of actions in 1.b) are part of the equilibrium path (Q.E.D.).

This result tells us that the firms concluding the FA are those closest to any CA. In fact, what matters for each firm is her competitive advantage over other competitors in serving one specific CA only. In fact, at the first stage firms anticipate that the competition for the specific contract taking place at the second stage will be *de facto* a Bertrand competition. A key point is that when the first stage offers are submitted there is uncertainty about which one, among M possible Bertrand games, will be effectively played in the second stage, depending on which CA will make the call-off. In spite of this uncertainty, Lemma 1 shows that each competitor focuses on one of these possible games only. This mechanism triggers extremely tough competition at the first stage.

WELFARE ANALYSIS

In our starkly stylized framework, given the assumption of complete information, the most efficient way for the CAs to purchase the good should be to call for tenders autonomously. In fact, this would guarantee that the closest firm would win the contract by simply offering the cost of the second-closest firm. Such an outcome would thus assure maximum efficiency (all the CAs would always be served).

Nevertheless, in what follows, we assume that it is not possible or profitable for the government to award each contract by allowing every single CA to procure the supply autonomously (i.e. without availing itself of the framework agreement concluded by a central purchasing body). This may be due to several reasons, such as legal constraints, reduction of the total process costs of competitive tenders, limited skills of single authorities in managing tenders, or reduction of the risk of corruption.

We then focus on the case of a CPB in charge of concluding a framework agreement. Since typically public sector's savings are the main concern of a CPB, a natural assumption about its preferences is a utilitarian wealth function, based just on CAs' utility (and not on vendors' utility). Thus, let u_j denote the j-th CA's utility. The CPB's objective function can be written as

$$U(n) = \sum_{j=1}^{M} \pi_j(n)\left(V - c_j(n)\right)$$

where π_j is the probability for the j-th CA of being supplied in the event it makes a call-off.

As it should be clear from Proposition 1, given the market configuration (N,M), both the probability of being supplied and the price to be paid by each CA depend on the number of winners of the FA, n. Since from the CPB's viewpoint the market structure is given, n is the only variable the purchasing body is able to control. Therefore, the CPB shall set n in order to maximize $U(n)$. By internalizing the previous results, given V, N and M, the CPB is able to compute its expected utility for each number of winners of the FA.

An important implication of Lemma 1 is that the number of potentially supplied CAs is exactly n, since the FA is concluded with n firms and each of them only targets one CA. As a consequence, as long as n<N, some CAs have no positive probability of being supplied, even in the case they make the call-off. It is easy to guess that these are the CAs the farthest from a supplier. Conversely, those CAs closest to a firm are eventually supplied with probability 1.

More formally, for each CA, the probability of being supplied can be computed in virtue of the result of Proposition 1. Let $i(j)$ denote the closest firm to the CA j, and let v_n and $v_{<n}$ denote the number of firms whose d_i is equal to and lower than d^n, respectively. Then:

$\pi_j=0$ if $i(j)$ is such that $d_i>d^n$

$\pi_j=(n-v_{<n})/v_n$ if $i(j)$ is such that $d_i=d^n$

$\pi_j=1$ if $i(j)$ is such that $d_i<d^n$

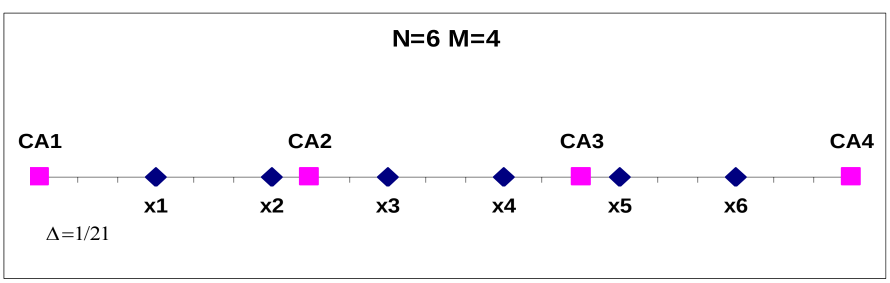
Now, since d^n is trivially a (weakly) increasing function of n, it is easy to realize that the same holds for π_j. This simply means that the higher n, the higher the probability for the CAs of being supplied. This improvement in efficiency clearly yields a positive effect on the first term of the CPB's utility function, $\pi_j(n)V$.

On the other hand, a higher *n* increases the supply cost as well. Indeed a higher *n* weakens the competition at the first stage, since the highest bid required for the firms to undercut at least *n* competitors becomes less aggressive, as shown in Proposition 1. As a consequence, at the second stage the constraint imposed by the first stage bid will be less stringent, thus making firms able to extract more surplus from trading. This effect leads to a decrease in the social utility. In other words, the CPB faces a clear trade-off between improving efficiency (in terms of likelihood that CAs are served) and improving competition (inducing higher savings on each single contract).

As we have already seen in Example 2, the solution of the trade off crucially depends on the value *V* of the good/service provided. When the value of the supply for the CAs is high, the most efficient choice is supplying as many CAs as possible, despite an increase in supply costs. Indeed, as it can be seen from the social utility function, a higher *V* strengthens the positive effect of *n* on the probability of trading, without affecting the cost.

In spite of the simplicity of our framework, the analytical characterization of the optimal *n* would lead to tedious calculations and formalism. This is due to the complex form of the functions $\pi_j(N,M,n)$ e $c_j(N,M,n)$, which are difficult to describe through elementary functions.

Yet, as we have already pointed out, for a CPB facing the optimization problem the market structure is known and given, and the set of possible *n* is generally quite limited. As a consequence, the solution of the problem for given *N* and *M* can be easily solved trough simulations. Thus here we will just illustrate a simulation for a given configuration *(N=6, M=4)*. In this case, $\Delta=1/21\approx0,29$.



By making use of the results of Proposition 1 one can easily determine the outcome of the tender and hence compute the social welfare function $U(n)$:

$$U(3) = U(4) = 2(V - 2\Delta) = 2V - 4\Delta$$

$$U(5) = 2 * 1/2(V - 3\Delta) + 2(V - 2\Delta) = 3V - 7\Delta, \text{ if } V \geq 3\Delta$$

$$U(5) = U(4), \text{ otherwise}$$

$$U(6) = 2(V - 6\Delta) + 2(V - 4\Delta) = 4V - 16\Delta, \text{ if } V \geq 6\Delta$$

$$U(6) = U(4), \text{ otherwise}$$

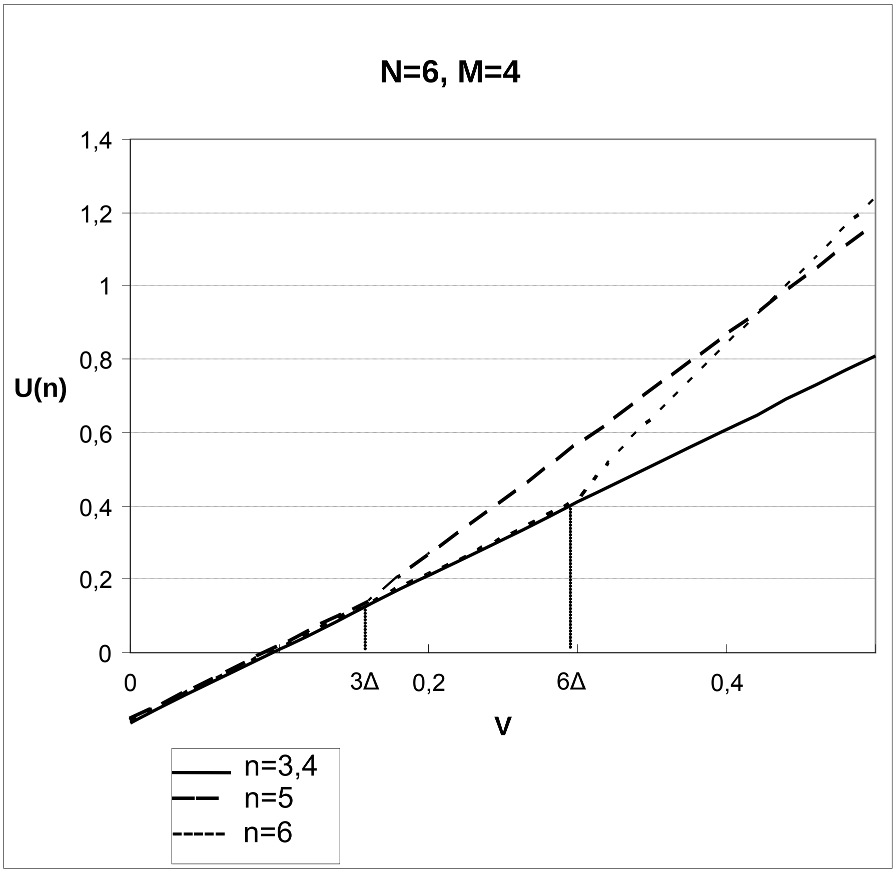
Figure 1 shows CPB's utility as a function of V, for different values of n. The reasoning is as follows.

From the point of view of social welfare, nothing changes between cases $n=3$ and $n=4$. When $n=3$, firms 2 and 5 have to undercut their most efficient competitors at the first stage, by offering $b_2=b_5=2\Delta-\varepsilon$. When $n=4$, first stage bids $b_i=3\Delta-\varepsilon$, $i=2, 3, 4, 5$ allow the four most efficient firms to qualify with certainty, since by doing that they can cut off the N-n less efficient competitors (firms 1 and 6). Nonetheless, Bertrand competition occurring at the second stage (provided that the call-off is made by CAs 2 or 3) yields an outcome equal to the case $n=3$. This is because both the most efficient firms (namely firms 2 and 5) and the second most efficient firms (namely firms 3 and 4) target the same CA (firms 2 and 3 target CA 2, firms 4 and 5 target CA 3), so that competition among them simply moves from stage one to stage two as n changes from 3 to 4.

From the CPB's viewpoint setting $n=5$ always dominates $n=4$. Indeed, the choice $n=5$ allows either firm 1 or 6 to conclude the FA, so that CAs 1 and 4 can be served (with probability $1/2$) if they make the call off. Remark that this does not affect the competition for serving CAs 2 and 3. The welfare improvement occurs because i) a new CA can be served; and ii) competition for entry between firms 1 and 6 pushes their first stage bids down to their supply cost. This analysis, however, holds if $V \geq 3\Delta$, for otherwise CAs 1 and 4 would never find convenient to purchase the good/service.

Finally, if $n=6$, no competition occurs at the first stage, so that firms 1 and 6 can exploit their competitive advantage with respect to their closest competitors in serving CAs 1 and 4, respectively. This

FIGURE 1
Social Welfare as a Function of V and for Different Values of *n*



affects CPB's utility only if V≥6Δ. If this is not the case, firms are able to extract the entire surplus from the CAs by charging a price $B_i=V$, where *i= 1, 6*. This configuration is welfare improving to the extent that the "higher efficiency" effect (now all the CAs are served with probability one) dominates the "lower competition" effect (now CAs 1 and 4 are served at higher prices). It is easy to realize that this only depends on the value of *V*. The relevant condition becomes *U(6,V)≥U(5,V)*, which holds when *V≥9Δ*.

CONCLUDING REMARKS

We have analyzed a simple model of Framework Agreement concluded with *n≥3* economic operators and with not all the terms of the contract laid down in the agreement. We have argued that the main rationale for using such a procurement tool is to satisfy the needs of different CAs whose preferences are heterogeneous with respect to the "incomplete" part of the contract. A clear trade-off emerges between fully satisfying the different CAs and triggering higher competition between firms, leading to higher savings.

We have abstracted away from entry costs (at the first stage). Although a detailed analysis is left for future research it is worth elaborating about how this variant may affect the model. First of all, extending the model by taking into account entry costs is a matter of realism, as very often participation costs represent a concrete barrier to entry into the public procurement market. Indeed, even besides cases where an entry fee may be imposed by the central purchasing body, submitting an offer may require by itself a considerable effort, due to the high complexity of the technical requirements of the contract.

In our stylized model this is a potentially interesting aspect, since the supply costs are modeled in terms of transportation costs (or distance from CAs' preferences) only. In other words, production costs are supposed to be equal among firms, and normalized to zero. The latter assumption is plausible when the production cost is a sunk cost, which is a cost that has been already paid by the firms when they face the entry decision. If this is not the case, the introduction of a fixed entry cost may be interpreted either as the cost of effectively participating in the tendering or as a pure production cost.

The model is sensitive to the introduction of a fixed entry cost, no matter how small. This is crucially due to the fact that, in order to enter the competitive process, sellers incur in no loss if entry cost are nil. As a consequence, they are willing to compete very aggressively to reach the second stage, independently of how large is their expected payoff, until the latter reduces to zero.

The key point is that bids submitted by competitors who get zero profit play a crucial role in identifying the SPE of the model. This is the main intuition explaining why the introduction of an arbitrarily small entry cost dramatically changes the nature of equilibria. In fact, as some competitors prefer to draw back from the competition without bidding (and everyone always will, if she has no chance of making positive profit), other (more efficient) firms are willing to increase their bid so as to increase expected profit. However, as the entrants' bid is high enough to make the entrance for the outsiders profitable, the latter find themselves potentially in a position to submit a bid that would ensure entry. Such a simple reasoning rules out any pure strategy equilibrium so that, in general, only mixed strategy SPE will exist.

NOTES

1. We are grateful to F. Dini, N. Dimitri and R. Zampino for useful inputs.
2. "Where a framework agreement is concluded with several economic operators, the latter must be at least three in number, insofar as there is a sufficient number of economic operators to satisfy the selection criteria and/or of admissible tenders which meet the award criteria."
3. The set of winners of the FA, *I,* may be not completely identified by the vector of bids *b* because, in case of tie, some winners are selected randomly on the strength of the tie-breaking rule.
4. The first assumption is standard, and it is needed in order to prevent non-existence of equilibria in games with continuous sets of actions. The second assumption will just simplify the description of the equilibria from a computational point of view, without affecting the results.
5. Notice that they could also chose to make no offer, so that $B_2=B_3=\varnothing$. Yet, in this case, despite the equilibrium strategic profile is different, the outcome of the game is the same.
6. Since all the competitors qualify for the second stage, at the first stage no competition occurs. Formally, for each firm, any bid $b\geq 2\Delta$ in the first stage may be part of an equilibrium strategy.

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