Chapter 15

A PROCUREMENT AUCTION MODEL FOR FRAMEWORK AGREEMENTS

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INTRODUCTION

Governments face sizable costs when procuring products and services from private companies. A common procurement mechanism used by governments is competitive auctions. Auctions have been thoroughly studied for many years in the economics and industrial engineering literatures. Framework agreements (FAs) are another procurement mechanism that is used in practice. FAs have received much less attention in the academic literature (See Albano and Sparro [2010] for an overview of the design issues that arise in FAs, and Albano and Sparro [2008] for one of the few economic modeling papers that study FAs).

Moreover, given the inherent uncertainty that arises in FAs, their analysis is likely to be more involved than that of standard auctions. Uncertainty arises, for example, because the spot market price of the products in the FA may change over the course of the contract, and the demand for the products in the FA is generally unpredictable.

The objective of this work is to build an economic/mathematical model to study the behavior of the buyers and sellers in FAs and use the results to suggest improvements to the design of them. In this paper, we focus on the option value aspect of FAs. More specifically, in a FA the price of a product or service is set at the beginning of the time horizon and the seller has the obligation to sell at that price. In that sense, the FA provides an “option” to buy at a pre-determined price over a time horizon; the spot market price could change over that sense, the FA provides an "option" to buy at a pre-determined price over a time horizon; the spot market price could change over that horizon. Indeed, if the spot price goes down, the buyer may decide to buy from the spot market as opposed to buying from the FA. An important objective of our research will be to study and asses this "option value" for the buyer as well as the "cost" induced for the sellers. In this chapter, we briefly explain the context and motivation of this research, introduce a simple model to study these issues, and summarize the main results. The remaining part of the chapter will provide the technical analysis. See also Gur, Lu, and Weintraub (2012) for more details and extensions.

MOTIVATION

Dirección Chilecompra is the national procurement agency of Chile. Its mission is to manage the system by which government entities carry out their procurement processes. The buyers can execute their procurement using different methods; the most important are procurement auctions, and framework agreements.

In a FA, the suppliers are selected by means of a public tendering process (managed by Dirección Chilecompra) in which the winners get the right and obligation of providing specific products in a web store called Chilecompra Express, for a given period. The price of the products and other conditions are defined in the mentioned tendering process. Then, all the government entities can buy from the selected suppliers directly (without a public tendering process).

In the Chilecompra System, in 2011, US $1.5 billion were traded using FAs (19% of the total amount traded in the system). By December 2011, there were 52 framework agreements, including more than 120.000 products and services. The product categories are diverse, ranging from food to software development services, the more relevant (dollar amount) being dialysis services, meal vouchers, computers and related devices, and vehicles.

For Dirección Chilecompra (as a Procurement Authority) it is important to obtain knowledge about the behavior of the agents participating in FAs. One particular issue is the existence of prices higher than those of the "general open market" in some product categories. It was decided to explore the forces that impact the behavior of the suppliers in FAs. The first force to be analyzed is the effect of price locking at the beginning of the FAs.

BASIC MODEL

Our basic FA model is built on the classic modeling approach in auction theory (Milgrom, 2004; Krishna, 2010). In this framework, the buyer proposes a FA mechanism that induces a game of incomplete information between sellers. The incomplete information arises, for example, because each bidder has a private cost (only known to itself but not to its competitors) for the products being sold in the FA. The private cost could be driven by private information each bidder has on its own technology, logistics, production costs, capacity, etc. In addition, we incorporated uncertain quantities, such as future demand or future spot prices by representing them with random variables, whose realization is unknown to the bidders and to the planner of the FA at the beginning of the time horizon. In this way, our model generalized standard auction models to incorporate the complexities that arise in FAs.

Specifically, the basic FA model considers the following elements:

- The government or buyer wants to buy one unit of one good at time period $t = 1$.
- There are N risk neutral bidders or sellers that, at time period $t = 0$, participate in a first price auction to obtain the *right* of selling the good. There is one winner in this auction.
- Each bidder has a cost $c_i + X$ of providing the good.
- The random variables c_i are a private information cost for each bidder, that are independent and identically distributed (i.i.d.) with distribution function F, finite mean μ_c, continuous density function f, and support $[\underline{c}, \overline{c}]$. At the moment of the auction ($t = 0$), bidder i knows its own cost c_i, but doesn't know the costs c_j of its competitors. From the perspective of bidder i, the costs c_j of its competitors are i.i.d realizations according to F.
- The random variable X is common to all bidders and at $t = 0$ its realization is unknown to all bidders who only know that it is drawn from a distribution function G with finite mean $E(X)$, and

continuous density function g. The random variables c_i and X are independent.

- At $t = 1$, the buyer has the option to buy from the FA at the price agreed in the first price auction at $t = 0$ or he can buy from the spot market at a price $C + x$, where x is the actual realization of X that has been revealed at $t = 1$, and C is a constant. The buyer buys from the spot market if it is more convenient.
- All components of the model other than the realized values $c_i's$ are assumed to be commonly known to all bidders. In particular, the distributions F; G are common knowledge, as is the number of bidders N and the constant C.

The model deserves several comments. First, we summarize and interpret some of the quantities introduced above. The cost component c_i is private and idiosyncratic and we interpret it as being associated to particular characteristics of the firm, such as, its managerial ability, logistics costs, etc.. On the other hand, the cost component X is common, and we interpret it as being related to the price of inputs that is common across all firms. It is random, because this price may change from the moment when the auction is run ($t = 0$) until the actual good in the FA is demanded ($t = 1$).

Second, the buyer needs the good at $t = 1$. At that point in time, he can buy from the FA or from the spot market. The price in the latter is $C + x$. The idea is that the government can buy from a supplier in the spot market that charges a price that is the actual realization of X plus some constant C. In general, this constant C will be related to the cost component c_i. For example, a common assumption will be that $C = \mu_i$, so the spot market supplier charges a price equal to the actual realization of X plus the average of the cost component c_i. In this case, in the spot market the buyer has access to an “average supplier”. We note that the constant C may also incorporate a “mark-up” on top of the average cost.

Third, in the basic model the winning supplier in the auction is not allowed to reduce its price to match the spot market price at $t = 1$.

Fourth, we have obviously abstracted away from many of the complexities that arise in real-world FAs, such as the existence of many products, many winners, other type of uncertainties, and so forth. The idea is to focus only on what we believe is essential to understand the issue at hand: the simplest model that highlights the unique characteristics of Framework Agreements. As we will see, even this simple model is hard to analyze.

SUMMARY OF MAIN RESULTS

In this section, we summarize the main results at a conceptual level omitting technical details.

In the process of analyzing the FA model we start by solving a simpler first price auction model that serves as a benchmark for the FA model. We also study different variants of a FA. The technical analysis is provided in the following section. There, we provide expressions in the form of integral equations that characterize the Bayes-Nash equilibrium (BNE) bid functions of the different models. We also provide ordinary differential equations (ODEs) whose solutions constitute the BNE strategies; we solve these ODEs numerically (see section "Numerical Experiments").

We study the following models:

1. As a benchmark, we analyze a first price auction that is conducted when the need arises at t=1, and where the buyer has the option of buying from the spot market. This model is equivalent to a FA model in which the prices are updated according to a perfect price index.
2. A basic FA as described above in the Section "Basic Model".
3. A basic FA in which the winning bidder can declare itself out-of-stock if the random part of the cost has a high enough realization, so that a sell would result in a loss for the winning bidder.
4. A "flexible" FA where, if its initial bid is higher than the spot market price, the winning bidder is allowed to match it.

The insights we obtained, which we summarize below, follow from the theoretical expressions derived as well as from the solutions of the numerical experiments.

First, our results suggest that, generally, the basic FA (2) induces higher bids than does the first price auction (1). The intuition is that in the FA, suppliers face risk with respect to their costs and they charge for it.

Second, compared to the standard FA model (2), the flexible FA (4) introduces two opposing forces. On one hand, the ability to match the spot market price relaxes the aggressiveness of bids, increasing bid prices. This is because now bidders do not need to overcome the spot market price ex-ante; they can always match it ex-post. On the other hand, conditional on winning, bidders face a more convenient environment, and therefore, bidders compete more intensely to win. In summary, allowing the bidders to match the spot market price relaxes the competition with the spot market price, but intensifies the competition with other bidders. We conjecture the bid prices may be larger or smaller than the ones in the original FA model depending on which of these two effects dominates. In our numerical experiments, the second effect dominated, and we observed lower bids in (4) than in (2).

Note that, generally, higher bid prices will result in smaller chances of executing the option, that is in smaller chances of buying from the winning bidder as opposed to from the spot market. In addition, generally, higher bid prices will result in higher overall expected prices.

In terms of design of FAs, our results suggest the following prescriptions:

1. Designing FAs which include price updating mechanisms based on appropriate price indexes can result in significant savings for the governments via more competitive bids.
2. Designing FAs that are more convenient for bidders, for example, by providing the flexibility of matching the spot market price, can encourage competition and reduce prices.
3. However, too much flexibility can also hurt. We recommend that the government avoid a design in which winning bidders can declare themselves out of stock, so that their bids are not really a commitment.

For the procurement authority (as the FAs management entity) these prescriptions should be translated into a series of initiatives to be implemented and some challenges could arise. Although the specifics of these issues depend on the particular public procurement framework, we will address these considerations briefly, in a broad sense.

If possible, the authority should consider, in the design of FAs, rules to update the price of the products, according to some “price index”. While for some line of products such as commodities this may be simpler to implement, for other products for which natural prices indexes are not available, it may be more challenging. In the latter case, and in particular for large transactions, running immediate first price auctions should be evaluated as they may help reduce procurement costs.

Allowing a match with the spot market price (if the FA price is above the market) is a common practice and, in general, it should not be difficult to implement. In Chilean FAs this is the default behavior, and it fits well within the public procurement framework. As with every “flexibility”, the convenience of this mechanism should be analyzed with caution.

Ensuring that the bids are a commitment is a main concern, as this determines the usefulness and credibility of the FA mechanism. In this sense, if one found it impossible to prevent suppliers from declaring themselves “out of stock”, the suppliers should be allowed to do it only in very special qualified cases and then only after rigorous evaluation. This could be a challenge in terms of the effectiveness and efficiency of the procurement authorities in the role of supervisor and enforcer of the FA contracts.

We finish this section by concisely commenting on the limitations of our analysis and results. First, the integral equations and ODEs that we derive do not have closed form solutions. Therefore, we cannot characterize analytically the equilibrium bid functions of the different models; this introduces challenges when comparing bid functions. The comparisons that we provided above are derived by the insights the equations give and by the numerical results, and we think they are very useful. Additional results regarding the comparisons can be found in Gur, Lu, and Weintraub (2012).

Second, our numerical results are based on solving ODEs; these ODEs are not well behaved mathematically. Therefore, the numerical analysis needs to be done with caution as the solutions are sometimes unstable.

Third, the numerical results provide a prediction of the bids one may expect when bidders play the auction game. We think these results provide useful qualitative insights. However, we do not advise that the practitioner take these results literally in the sense that these are the bids one should expect in a real world FA. Our models do not incorporate many of the additional complexities that exist in practice. Hence, while we think our results highlight important first order effects and qualitative insights, we do not think they provide the actual bids bidders will submit in the real world.

ANALYSIS OF THE MODEL

First Price Procurement Auction, With Outside Option

To start we consider a first price procurement auction model that is run when the need arises at t=1, and where the spot market is an outside option to buy the product. It is simple to observe that from the auctioneer's perspective, this is equivalent to running a first price auction at t=0 with costs $c_i + E(X)$, and where there is no spot market price uncertainty.

Notably, this model is equivalent to a model of a FA in which the government provides a perfect price index that follows the random part of costs X. More specifically, the winning bid b^* received at t=0, is updated to $b^* + (X - E(X))$ at t=1, so that the actual profits for the winning firm are deterministic and given by $b^* - c - E(X)$. Note that because this price index follows the trajectory of X perfectly, it completely removes the common cost uncertainty for the winning bidder.

We study the symmetric BNE of this auction game by using a similar argument to the standard model (See, for example, Krishna, [2010] and Milgrom [2004]). First, we note that equilibrium must be in strictly increasing bidding strategies.

Now, the expected equilibrium profit of the bidder i is:

$$\pi_i(c_i) = \big(b(c_i) - c_i - E(X)\big)P(i\ wins),$$

Where $b(c)$ is the symmetric equilibrium strategy.

Note that bidders are risk neutral and in this case whether i wins is independent of X. Because $b(c)$ is a symmetric strictly increasing strategy, i wins if and only if i has the lowest cost. Since X is a common cost component, we can write

$$\pi_i(c_i) = \big(b(c_i) - c_i - E(X)\big)\overline{F}^{N-1}(c_i)$$

Where $\overline{F}(x) = 1 - F(x)$

We can also write

$$\pi(c_i) = \max_{b_i}\big(b_i - c_i - E(X)\big)P\big[b_i < b(c_j), j \neq i\big]$$

$$= \max_{b_i}\big(b_i - c_i - E(X)\big)\overline{F}^{N-1}(b^{-1}(b_i)) \quad (1)$$

By the envelop theorem

$$\frac{d\pi(c_i)}{dc_i} = -\overline{F}^{N-1}\Big(b^{-1}\big(b(c_i)\big)\Big) = -\overline{F}^{N-1}(c_i)$$

Integrating, and realizing that because of the competition with the outside spot market option a bidder with cost μ_c or more will never win the auction, we get:

$$\pi(c_i) = \int_{c_i}^{\mu_c} \overline{F}^{N-1}(c)dc$$

Hence, the symmetric BNE strategy is given by:

$$b(c_i) = c_i + E(X) + \frac{1}{\overline{F}^{N-1}(c_i)} \int_{c_i}^{\mu_c} \overline{F}^{N-1}(c)dc \qquad (2)$$

Ordinary Differential Equation

Assuming that its competitors $j \neq i$ use the identical bidding strategy $b(c_j)$, the expected profit function of bidder i is given by (see equation (1)):

$$\pi(b_i) = \big(b_i - c_i - E(X)\big)\overline{F}^{N-1}(b^{-1}(b_i))$$

Taking first order condition (ignoring the subindex i to simplify notation) we get:

$$\overline{F}^{N-1}\big(\phi(b)\big) - (b - c - E(X))\ (N-1)\ f(\phi(b))\phi'(b)\overline{F}^{N-2}(\phi(b)) = 0$$

Where we have defined $\phi\ (b_i\ ) = b^{-1}(b_i)$

Now, dividing by $\overline{F}^{N-2}\big(\phi(b)\big) \neq 0$, and using that in a BNE $b = b(c)$, so that $\phi(b) = c$, and that $\phi'(b) = 1/b'(c)$:

$$\overline{F}(c) - \big(b(c) - c - E(X)\big)(N-1)\frac{f(c)}{b'(c)} = 0$$

and therefore we get the ODE:

$$b'(c) = \big(b(c) - c - E(X)\big)(N-1)\frac{f(c)}{\bar{F}(c)}$$

with the boundary condition

$$b(\mu_c) = \mu_c + E(X).$$

The boundary condition establishes that the bidder with the highest possible cost bids its cost, so he does not make money even when winning. We can solve this ODE numerically. See Hubbard and Paarsch (2011) and Fibich and Gavish (2011) for good summaries of the challenges involved numerically solving BNE strategies using ODEs.

If we take $c \sim U\,[0,\alpha]$ and $X \sim U\,[0,\beta]$, we have for $c \in [0,\alpha]$:

$$b'(c) = \left(b(c) - c - \frac{\beta}{2}\right)(N-1)\frac{1/\alpha}{1 - c/\alpha} = \left(b(c) - c - \frac{\beta}{2}\right)\frac{N-1}{\alpha - c}$$

Basic FA Model

Here we analyze the FA model introduced in section "Basic Model". Note that, unlike the previous model, here bidders have costs $c_i + X$ where X is random and realized at the second stage. We note that this model is equivalent to an auction in which the winner competes against an outside market alternative in the second stage, but cannot lower his bid if he is defeated by the market price.

Following the same approach as in the previous models we get the integral equation:

$$b(c_i) = c_i + \frac{E\big[X\mathbb{I}_{\{b(c_i) < \mu_c + X\}}\big]}{\bar{G}(b(c_i) - \mu_c)} + \frac{1}{\bar{F}^{N-1}(c_i)\bar{G}(b(c_i) - \mu_c)}\int_{c_i}^{\mu_c} \bar{F}^{N-1}(c)\bar{G}(b(c) - \mu_c)\,dc$$

By looking at the integral equation that characterizes the equilibrium bids, we note that the FA may induce larger equilibrium bids than the first price auction. The intuition is that in the FA model the bidder wins in the scenarios when the cost component X is high, when the supplier is expensive. Therefore, he needs to charge for this. An alternative intuition, is that whenever bidders can "lock" their variable cost by a risk-reducing opportunity, they are able to compete more aggressively. Again, the above integral represents a non-linear differential equation with no known closed form solution. The following is the respective ODE that we solve numerically. Taking $c \sim U[0, \alpha]$ and $x \sim U[0, \beta]$, we have:

$$b'(c) = \frac{(N-1)\left[(b(c)-c)\left(\beta - b(c) + \frac{\alpha}{2}\right) - \frac{\beta^2 - \left(b(c) - \frac{\alpha}{2}\right)^2}{2}\right]}{(\alpha - c)\left(\beta + c - b(c)\right)}$$

With the boundary condition $b(\alpha/2) = \alpha/2 + \beta$.

FA Model with Out-of-Stocks

In this Section we consider a model in which whenever the realized cost X in $t = 1$ is high enough such that the total cost of the winner is higher than its bid, the winner can declare himself "out of stock" in $t = 1$ thus avoiding the loss incurred by such a sell. To be precise, whenever

$$b^* < c^* + X$$

Where $c^*$and b^* are the cost and bid of the winner, there is no sell in $t = 1$. Hence a bidder with bid $b(c)$ that is more competitive than its competitors will make a sell if two conditions are satisfied (for $c \leq \mu_c$):

$$c + X \leq b(c) \leq \mu_c + X$$

The first one relates to the "out of stock" condition mentioned above. The second, as before, requires that the winning bid is smaller than the spot market price.

One can derive an integral equation and a respective ODE for this model. We note that whenever the costs c and X are uniformly distributed with $c \sim U\ [\alpha_1, \alpha_2]$ and $X \sim U\ [\beta_1, \beta_2]$, the profit function is independent of the bid, and the above differential equation is not well defined. Hence, we cannot solve this ODE numerically.

Flexible FA: Allowing the Bidders to Match Spot Market Price

Consider a case in which the lowest bidder (i.e. the winner of the first stage) is allowed to lower his bid after the realization of X in the second stage $t = 1$ to match the spot market price $\mu_c + x$. We note that with the realization of X, at that point, the spot market price is known to the winner of the first stage.

Compared to the original FA model (Basic Model), there are two opposing forces in the equilibrium bids:

- On one hand, the ability to match the spot market price at $\mathrm{t} = 1$ relaxes the aggressiveness of bids at $\mathrm{t} = 0$, increasing bid prices. This is because now bidders do not need to overcome the spot market price ex-ante, they can always match it ex-post.
- However, conditional on winning, bidders face a more convenient environment. We can observe this by comparing the profit function of this model with the one in the basic model; they coincide except for an additional term in the former given by the flexibility of matching the spot market price. This makes bidders compete more intensely to win.

In summary, allowing the bidders to match the spot market price relaxes the competition with the spot market price, but intensifies the competition with other bidders. We conjecture the bid prices may be larger or smaller than the ones in the original FA model depending on which of these two effects dominates. We also derived the following ODE, and studied it numerically. Taking $c \sim U[0, \alpha]$ and $x \sim U[0, \beta]$:

$$b'(c) = \frac{(N-1)\left[\left(b(c) - \frac{\alpha}{2}\right)\left(\beta - b(c) + \frac{\alpha}{2}\right) + \beta\left(\frac{\alpha}{2} - c\right) - \frac{\beta^2 - \left(b(c) - \frac{\alpha}{2}\right)^2}{2}\right]}{(\alpha - c)\left(\beta - b(c) + \frac{\alpha}{2}\right)}$$

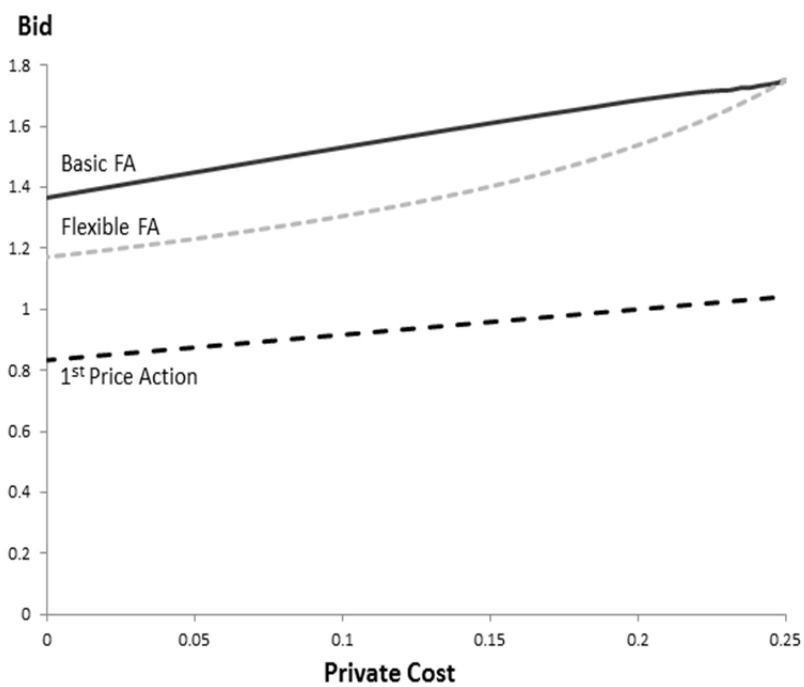
NUMERICAL EXPERIMENTS

In this section we used the ODEs derived in the previous section to numerically derive the equilibrium bid functions of each of the models presented. In the figures we present a set of representative results where we present the equilibrium bid functions for the first price auction, the basic FA, and the flexible FA. We do not present the results for the out-of-stocks FA, because the respective ODE is not well defined for uniform distributions. In the experiments, we assume the random variables c_i are $Uniform[0, \alpha]$ and the random variable X is $Uniform[0, \beta]$.

In addition, we use the following parameters: $\alpha = 0.5$, $\beta = 1.5$, $N = 6$ (number of bidders). Note that, as suggested by our ODE theory, the bid functions can only be plotted up to μ_c which in this case is $\alpha/2 = 0.25$. The ODEs are solved using a Runge-Kutta method in Matlab.

In Figure 1 we can see that the results are as expected and as suggested by the findings discussed in the previous section. First, the first price auction (or perfect price index model) induces the lowest and most competitive bids. Second, the basic FA induces the largest bids. Finally, the flexible FA (with spot market matching) induces bids that are in between. Therefore, in this case, the effect of additional competition with other bidders induces lower bids compared to the standard FA.

FIGURE 1
Numerical Comparisons between Models



One can use these ODEs to obtain results for other parameters and specifications. This needs to be done with some caution, however, because as mentioned before, the ODEs are poorly behaved at the boundary conditions. Sometimes, this introduces instabilities in the solution of the ODEs (See Hubbard and Paarsch [2011], and Fibich and Gavish [2011] for good summaries of the challenges involved numerically in solving BNE strategies using ODEs).

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