DESIGN OF REASONABLE MULTIDIMENSIONAL AUCTIONS UNDER REGULATOR IGNORANCE

Pier Angelo Mori

ABSTRACT. The paper's basic question is how a non-Bayesian regulator is to design multidimensional franchise auctions in the absence of a prior on firms' costs and market demand. A first result is the characterization of the environments which admit prior-independent optimal second-score auctions as those where bidders have equal marginal costs. An implication of this result is that no prior-independent optimum exists over the universal domain of priors and hence a regulator who has no information at all about firms' costs is bound to resort to weaker choice criteria than prior-independent optimality. However, even non-Bayesian regulators may have available information that helps restrict the prior domain, as is the case for some public services which display equality of marginal costs across potential contractors. In this restricted domain we are able to prove the existence of (constrained) prior-independent optimal auction rules characterized by the coincidence of the score function with the social welfare one. A problem with this rule arises when social welfare reflects concern for consumer surplus, since this is not computable under complete ignorance about market demand and the choice of any computable score functions generally leads to non-optimal outcomes. Though a fully rational choice is precluded in this case, it is nonetheless possible to make a reasonable choice of the score function. A few minimal reasonableness requirements are discussed which are immediately applicable to regulatory practice.

INTRODUCTION

In this paper we make a first step towards a non-Bayesian theory of franchise-auction design. The fundamental problem for a regulator who ignores the probability distribution of environment parameters is that he is generally unable to identify the socially optimal auction. The exception is when there exist mechanisms that dominate every other irrespective of the prior. Our main aim here is to characterize the structural conditions under which such auction mechanisms exist.

Though our task is mainly theoretical, we eventually work out a few hints for regulatory practice too as a by-product of our analysis.

The regulatory problem on which we focus here is a direct generalization of Riordan and Sappington's (1987) framework, which is also the basis of a few franchise-award models such as Che (1993) and Branco (1997). A regulator is to award a public service contract whose economic variables are the tariff paid by customers and the subsidy paid to the contractor by the regulator. In these models the regulator is uncertain only about marginal costs while fixed costs are taken to be common knowledge. Such a restriction is clearly artificial and just aimed at removing the well-known difficulties of the multidimensional case: as a matter of fact there is no reason why the principal should be perfectly informed about fixed costs while he is uncertain about marginal ones (Laffont, Maskin e Rochet, 1987; McAfee e McMillan, 1988; Armstrong, 1996; Rochet e Choné, 1998; Armstrong e Rochet, 1999), and all the more so in the field of public services where fixed costs are often the main source of cost variability across potential contractors. In this paper we assume the regulator to be uncertain about both. A further unsatisfactory feature of these models is that the regulator's uncertainty is restricted to the supply-side of the market by assuming that demand functions are common knowledge. This assumption seems quite unrealistic, since demand for many services – especially new ones – is not at all easier to estimate than costs: here we assume that the regulator is uncertain about market demand too. Finally, contrary to the Bayesian approach, we also assume that the regulator has no priors on cost and demand functions.

Our interest is in the design of multidimensional auctions where bids consist of many elements. These are frequently used in practice for the award of complex contracts which involve several variables, as is typical, though not exclusive of public service contracts (tariffs and subsidies are prime examples of such variables). There is a growing literature on multidimensional auctions which originates from Che (1993). A few papers like Bushnell and Oren (1995) and Asker and Cantillon (2004) have extended the analysis to environments with multidimensional adverse selection. The main difference to these contributions is that here we have an explict design problem with a non-Bayesian regulator: this paper's basic question is how an ignorant regulator is to design a franchise auction when the choice is restricted to multidimensional ones.

We develop two main sets of results, the first of which concerns second-score auctions. We start by asking whether in these circumstances there exist prior-independent optimal auctions – i.e.

auctions that dominate every other irrespective of the prior – in the class of second-score ones. This essentially amounts to asking if a prior-independent optimal scoring rule exists for use in second-score auctions. When variable costs display the single-crossing property, it is indeed possible to characterize the environments which admit prior-independent optimal (pointwise-optimal) second-score auctions. In section 3 we prove that such optima exist if and only if variable-cost parameters are equal across all bidders and that these are simply identified by a score function which coincides with the social welfare one. This characterization result for second-score auctions turns out useful to cast light on the existence of optima in a wider context. If marginal and fixed costs are i.i.d. across potential contractors, all multidimensional auctions with the same score function are revenue-equivalent (Asker and Cantillon, 2004). Then if a prior-independent optimal score function exists, it must also be optimal with respect to i.i.d. cost parameters and therefore a prior-independent optimal second-score one must exist even when marginal costs differ across firms: since our characterization result rules this out, we can conclude that such optimal auctions do not exist over the universal domain of prior distributions.

The impossibility result opens up two enquiry avenues. One is to relax the prior-independent optimality criterion, which indeed proves too strong in this context, and opt for weaker ones that can be sensibly applied to the problem at hand. The other one is to keep the criterion but apply it to restricted domains. The point is that in specific circumstances such as the award of public-service franchises a non-Bayesian regulator may have available information that allows to delimit the problem suitably and find a solution even under the strong prior-independence criterion. One such example in the field of public services is the property of equal marginal costs across potential contractors: many services like bus transportation, waste collection, etc., display this property and the regulator can be aware of it even if he has no prior on cost parameters. This piece of information in fact amounts to a restriction of the prior field (though not knowing the prior, the regulator knows that it belongs to a well-defined restricted domain) and surprisingly turns out enough to allow for a solution to the auction-design problem under the strong optimality criterion: the second set of results is about the design of franchise auctions in this specific set-up and according to the (constrained) prior-independence optimality criterion. If in such circumstances fixed costs are independently and identically distributed across bidders with equal marginal costs across bidders, there holds a revenue-equivalence result by which we are able to prove that both first-score and second-score auctions are optimal in expected value with respect to all priors and auction mechanisms if welfare functions are used to score bids (Proposition 2).

In order to implement an auction that is prior-independent optimal in the class of second-score ones the regulator's required minimal information is to know that variable-cost parameters are equal across bidders and that firms know this too. If in addition it is also known that fixed costs are i.i.d., the regulator, though ignoring the prior, will be able to identify the optimal solution in the set of all auction mechanisms. In both cases the optimal score function coincides with the social welfare function. Of course, to score bids according to his welfare function the regulator must know it, which is not always the case: if this is a weighted sum of consumer and producer surpluses, as is usually assumed in regulation theory, a regulator who has no information about market demand does not know it either. Then even under the equality of marginal costs across contractors the possibility of a fully rational choice by an ignorant regulator vanishes altogether. However, in our context one can devise satisfying, though not fully rational ways of solving the regulator's problem when demand ignorance is added to cost ignorance.

When a generic score function is used in a first- or second-score auction with equal variable-cost parameters across bidders, an objective is actually maximized that coincides with the score function itself. In other words, the regulator in fact behaves as if he pursued an objective different from his natural one. To stress the distinction we call this second objective pseudo-objective. Given the one-to-one correspondence of score functions and pseudo-objectives under the equality of marginal costs across firms, choosing a score function is thus the same as choosing a pseudo-objective. It is at this point that the notion of reasonable pseudo-objective comes in: if the regulator is unable to behave in a fully rational way (i.e. to optimize his natural objective), he can however try to behave reasonably, i.e. to optimize a reasonable pseudo-objective. What is meant by this will vary from situation to situation. Here we examine two minimal conditions that must arguably be met by any reasonable pseudo-objective: computability and the ranking of economic bid variables according to some clearly defined notion of economic cost or benefit. In section 4 we discuss two examples of pseudo-objectives. The first of them is not known to be in use and is proposed here as a possible solution to the auction design problem for a practically relevant class of situations. The other is drawn from experience and is an example of unreasonable score function according to the principles developed here.

The paper is organized as follows. After laying out the model in section 1, we study prior-independent optimal auctions in section 2. First we characterize auctions that are optimal in this sense in the class of second-score ones. Then a number of results are derived about optimal auctions under the restriction of equal marginal costs across firms. Section 4 discusses the consequences of ignorance about demand functions and a notion of reasonable auction design. In subsections 4.1 and 4.2 we apply theory to solve two design problems of practical interest.

THE MODEL

A regulator is to select a contractor for the provision of a public service. Firm i's cost of service supply y is $c(y, m_i) + F_i$, where F_i is the fixed cost, $0 < F_i < \infty$, and m_i is a variable-cost parameter which varies over some interval ($c(0, m_i) = 0$ for all m_i). These parameters are private information to the firm. We denote service tariff p and the (possibly negative) subsidy granted to the contractor s. Service demand at every p is $y(p)$ and the contractor's profit is

$$\pi(p, s, m_i, F_i) = py(p) + s - c(y(p), m_i) - F_i$$

Demand and cost functions are of usual shape (respectively decreasing and increasing in y) and continuous. The regulator's objective is to maximize social welfare which we assume to be specified in the standard fashion after Baron and Myerson (1982)

$$W(p, s, m, F) = CS(p) - s + \alpha\pi(p, s, m, F) \quad (1)$$

where $CS(p)$ is the consumer surplus and $0 \le \alpha < 1$. Moreover the regulator has no prior on cost and demand functions.

To select the contractor the regulator runs a multidimensional auction. By this we mean an auction where each competitor bids a contract (p, s), bids get scores $V(p, s)$ according to some score function $V : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$, the highest-score bidder wins and losers pay nothing. Score functions are chosen by the regulator and the choice is restricted to functions that are continuous and monotonic decreasing in both variables. A further element of auction mechanisms is the rule by which the contract to be implemented by the winner is determined. In the present analysis we focus mainly on two rules which have received particular attention in the literature - first-score and second-score auctions. Under the former the implemented contract is the winner's bid. Second-score auctions are extensions of second-price ones where the winner is free to choose any contract with a score at least as great as the second-highest one. Auction mechanisms also usually specify a reserve value for each bid variable. In view of the regulator's severe informational constraints assumed above, reserve values are here given and hence fall outside the scope of auction design. As a consequence, in this model the choice of an auction mechanism reduces to the choice of a score function and a contract-determination rule. Moreover, to simplify calculations we assume that, unless indicated otherwise, reserve values are infinite, as in Che (1993) and Branco (1997), and that all firm types always participate in the auction (cf. Riordan and Sappington, 1987; Che, 1993; Branco, 1997).

The basic structure of this model coincides with that of Riordan and Sappington (1987) and Che (1993) except in two aspects - that here we have two uncertain cost parameters instead of one and, above all, the regulator has no prior on cost and demand functions as e.g. in non-Bayesian regulation models after Averch and Johnson (1962). These models typically investigate the properties of the regulator-regulated firm relationship under given rules and do not address the problem of regulatory-mechanism design, which has instead been the main object of the Bayesian approach since Baron and Myerson (1982). In this paper we attempt to join together the two strands of literature by investigating conditions under which an ignorant regulator can cope with the design of franchise auctions. Besides being theoretically interesting on its own, the question is also practically relevant, since Bayesian models assume regulators to be endowed with probability priors about demand and cost functions that they often lack in the real world. Without them the regulator cannot generally be rational in the usual sense and Bayesian models turn out useless for practical purposes. But we shall see that, if firms' costs have certain properties, one can devise ways of solving the regulator's problem that, though not fully rational, are satisfactory in a sense to be specified.

Optimal multidimensional auctions

For analytical ease we split the study of auction design under ignorance into two parts: in this section we put aside uncertainty about demand function and analyse the case where the regulator is uncertain about costs only; in the next section we shall deal with ignorance about demand function too.

To start with let us focus on second-score auctions. A Bayesian decision-maker faces a straightforward problem: what he has to do is just to select the score function which maximizes expected welfare with respect to his prior distribution of m_i and F_i. For a non-Bayesian regulator the problem is much more difficult since he has no prior and is therefore unable to compute expected welfare and compare alternative mechanisms with respect to it. There is however one case in which this limitation is irrelevant and even a severely uninformed decision-maker can recognize a socially optimal score function, i.e. when it maximizes expected welfare for all possible priors. The applicability of this optimization notion is the main theme of this section. As we shall see, restrictive though this criterion is, the problem does admit solutions in a few economically significant environments. We proceed in two steps: first we work on social welfare functions that are independent of the state of the world, i.e. $W(p,s,m,F) \equiv \hat{W}(p,s)$, and characterize prior-independent optimal auctions in the class of second-score ones under this restriction; then we shall extend the results to different auctions and more general welfare functions.

Let us see a couple of preliminary results that will be useful in the subsequent analysis. The following is just an extension to our context of the standard characterization of second-price equilibria (the proof is a slight variant of Vickrey's theorem and is left to the reader).

Lemma 1. *In any second-score auction with a continuous and monotonic decreasing score function $V(\cdot)$ there exists a dominant equilibrium where each bid maximizes the bidder's score subject to the zero-profit constraint.*

A typical situation where the regulator's welfare cannot be a pointwise-optimal score function is illustrated in Fig. 1 where the implemented bid differs from the second-score one. If we use social welfare $\hat{W}(\cdot)$ to score bids, the implemented contract is a and the second-score bid is b in the state of the world to which the depicted p-curves refer. In the figure there is also a level curve V belonging to a score function under which the second-score bid is d and the implemented contract is a couple c such that $\hat{W}(c) > \hat{W}(a)$. In other words, in the state of the world represented in the figure $\hat{W}$ is dominated by another score function and hence is not pointwise optimal. It is to be noted that the mere existence of a curve like V in some state is not enough for concluding that $\hat{W}$ is dominated. Suppose that in the figure we had $p_1(a) = 0$ in place of $p_1(a) > 0$, i.e. that the iso-profit curve through a were the zero-profit one. Then there would actually be two equilibria – one with a and the other with b as implemented contract, both of which are full-information optima – i.e. contracts that could be indifferently chosen by the regulator if he had full knowledge of the state of the world: here the divergence between first and second-score bids in one state does not cause $\hat{W}$ to be dominated. Except in this special case, however, $\hat{W}$ is never pointwise optimal if there exists a curve like V in Fig. 1, which in turn occurs whenever implemented contract and second-score bid do not coincide.

When bidders have different trade-offs (i.e. iso-profit curves are not vertical translations of one another), they will reply differently to changes in the score function and one can exploit such differences to improve welfare in at least one state. In the example of Fig. 1 the winner has a lower marginal rate of substitution of s for p, i.e. requires less as compensating tariff increase for a given subsidy reduction than the second-ranked firm. Then, by shifting from $\hat{W}(\cdot)$ to a flatter score function like $V(\cdot)$ (i.e. by putting less weight on tariff), the second bidder is induced to substitute subsidy for tariff and the new iso-score touches a lower iso-profit curve of the winner. If variations are not too large we are able to get an improvement in welfare as in the figure, where the implemented contract shifts from a to c (note that here bidder 2 over-reacts, as it were, by "asking" for a sharper tariff increase, thus worsening the welfare level attached to the second bid, but this is not relevant to the regulator who is only interested in the actual welfare which is attached to the implemented contract). Of course, shifting to a score function with an iso-score curve like V will not be welfare-improving in all states of the world but to our purposes it is enough that this is true of at least one state: in the appendix we prove that the divergence between implemented contract and second bid is indeed always sufficient for this to occur. In conclusion, for the pointwise optimality of $\hat{W}$ there is necessary either the coincidence between implemented and second-score bid or the fact that the implemented contract is a full-information optimum, as is stated in the next lemma (see appendix for the proof).

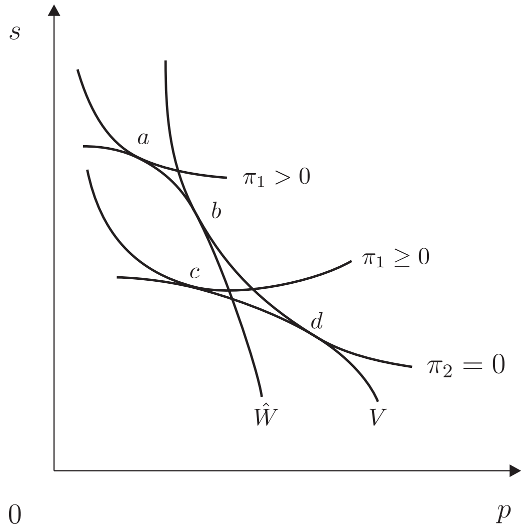

Figure 1

Lemma 2. *If in some state* $(m_i, F_i)_{i \in I}$ *the implemented contract under the social welfare function* $\hat{W}(\cdot)$ *differs from the second-score bid and is not a full-information optimum, the social welfare function is not a pointwise-optimal score function.*

Now we tackle the main question: are there solutions to the pointwise-optimization problem for second-score auctions? A sufficient condition for the pointwise optimality of $\hat{W}(\cdot)$ is given in the following lemma.

Lemma 3. *If variable costs are equal across firms for all production levels there exists a pointwise-optimal score function for second-score auctions which coincides with the regulator's welfare function.*

Proof

Consider any realization of the parameters $m, F_1, F_2, \ldots, F_n$, and without loss of generality assume that $F_i \leq F_{i+1}, i = 1, \ldots, n-1$. By Lemma 1 the second-highest bid belongs to bidder 2's zero-profit locus and also to the highest iso-score curve, as in Fig. 2. This is also the auction outcome since the winner, tough free to choose a contract different from (p_2, s_2), has no incentive to do so (the contract most advantageous to him among those with the second-highest score is just that): whenever bidders differ only in fixed costs and hence all firms' iso-profit curves have the same shape, the implemented contract and the second-highest bid always coincide. Now take any score function that at least in one state induces the second-highest bidder to choose a point on his iso-profit curve $\pi_2 = 0$ that does not maximize the regulator's welfare (if this were not true, the score function would be equivalent to $\hat{W}(\cdot)$). Then also the contract implemented under the new score function will entail a (weakly) lower welfare than that implemented under $\hat{W}(\cdot)$ (i.e. (p_2, s_2) of Fig. 2). In conclusion, by adopting a score function different from $\hat{W}(\cdot)$ the regulator obtains a (weakly) lower welfare in every $m, F_1, F_2, \ldots, F_n$. In other words, $\hat{W}(\cdot)$ dominates every $V(\cdot)$ as score function in every state.

Q.E.D.

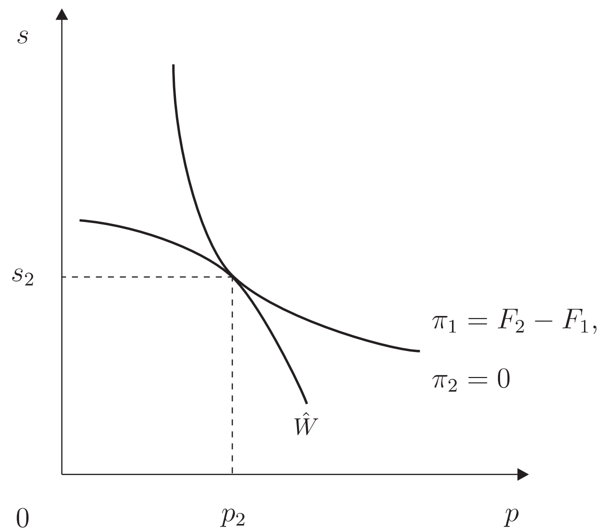

Figure 2

The intuition behind this result is very simple. When m_i is the same for every i, bidders differ at most in fixed costs (i.e. have the same isoprofit curves with a different scale, $\pi(p,s,m,F_i) = \pi(p,s,m,F_j) - F_i + F_j, \forall i, \forall j$). Then, since by definition the second-highest bid is the welfare-maximizing one subject to zero profit, in every state this also coincides with the highest-profit bid for the winner subject to the second score. Therefore a displacement of the score function from $\hat{W}(\cdot)$ would necessarily shift the second bid as well as the implemented contract to a lower-welfare position along the curve $\pi_2 = 0$.

This condition, however, is not necessary. Suppose, as in Fig. 3, that all isoprofit curves $\pi_1 \geq 0$ lie in the epigraph of $\pi_2 = 0$. Then, given any score function V the isoscore curve through the second bid has no intersections with the interior of the epigraph of $\pi_2 = 0$ and hence at no point on it profit can be larger than at a. If this holds true for every state of the world, there exists no V that strictly dominates $\hat{W}(\cdot)$ pointwise.

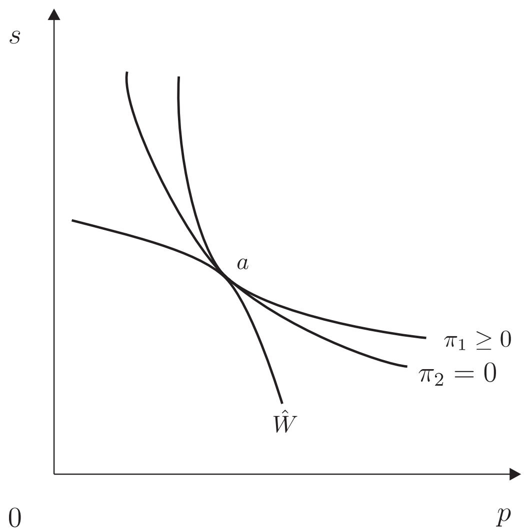

Figure 3

One remarkable condition under which the case of Fig. 3 cannot occur is the *single-crossing* property

$$\frac{\partial^2 c}{\partial y \partial \mu_i} > 0, \forall y, \forall \mu_i$$

which means that different types' iso-profit curves cross only once. With differentiable functions contract a has the characteristics of the figure only if the implemented tariff is such that $c'_\mu(y(p),\mu_1) = c'_\mu(y(p),\mu_2)$ and $c(y(p),\mu_1) - F_1 = c(y(p),\mu_2) - F_2$, but the single crossing property rules out the first condition for $\mu_1 \neq \mu_2$. Then in this class of environments there holds a straightforward characterization of pointwise optimal score functions (proof in the appendix).

Proposition 1. *In environments where cost functions display the single-crossing property there exists a prior-independent score function for second-score auctions if and only if variable costs do not vary across firms for all production levels, and this coincides with* $\hat{W}(\cdot)$.

Proposition 1 traces the boundary beyond which the search for a prior-independent (pointwise-optimal) optimal score function in second-score auctions fails: when the single-crossing property holds, such score functions exist in no other environments than those with variable-cost functions equal across firms.

We have seen that under the single-crossing property the regulator's problem has a complete solution in the class of second-score auctions when $m_i = m$ for all i and moreover this is the only case where it does. Now we address a different problem: is it possible to improve the regulator's welfare by resorting to other auction-awarding rules, like for example the first-score rule? The question has a straightforward answer when the regulator's objective is additively separable in subsidy, i.e. $\hat{W}(p,s) \equiv w(p) - s$. Assume fixed costs F_i are equally and independently distributed according to a generic distribution function $G(\cdot)$ with density $g(\cdot)$, which is common knowledge to firms, and moreover m and F_i are stochastically independent. Since all firms have the same m and the only uncertain parameters for them are their bidders' fixed costs, we can proceed in the usual way to calculate the expected value of a given score function with respect to $G(\cdot)$. That is, one first works out the winner's expected profit by applying the revelation principle. Then one finds by standard differential methods that any two auction mechanisms that induce the same ex-post allocation are equivalent in terms of expected score (for the proof see the appendix).

Lemma 4. ("*Revenue equivalence*") *With* $m_i = m$ *for all* i, m *and* F_i *stochastically independent and* F_i *i.i.d. all auctions with the same score function additively separable in* s *that award the contract to a firm with the lowest* F *and entail the same tariff outcome* $p^*(\cdot)$ *realize the same expected value of the score function.*

The lemma establishes the equivalence with respect to expected score of all auction rules that: i) are efficient (i.e. the lowest type F wins and the highest type has a null probability of winning), ii) induce the same tariff $p^*(\cdot)$ as outcome. Now note that both first- and second-score have just these two properties. Since parameter m_i is equal for all firms, the winner in both first- and second-score auctions is the bidder with the lowest F (the probability of winning is $(1-G(F))^{n-1}$). Moreover, given any separable score function of the type $V(p,s) \equiv v(p) - s$, in first- and second-score auctions the tariff is

$$p^*(m) = \arg\max_p \left[v(p) + py(p) - c(y(p),m) - F - \bar{V}\right]$$

for every m, irrespective of the score $\bar{V}$ to be met by the winner (this is actually true for any *n*-th score auction). Then by Lemma 4 first- and second-score auctions are equivalent in terms of expected score. Moreover, there is no other auction that can do better in expected terms than first- and second-score ones (or, for that matters, than any efficient auction with the same tariff outcomes): welfare functions additively separable in s dominate any other scoring criterion in first-score auctions for any distribution function $G(\cdot)$ too. To see this, suppose ex absurdo that there exists a score function that obtains a higher expected welfare under the first-score rule. By the equivalence result just established dominance should also hold under the second-score rule but this would contradict Lemma 3, which asserts the reverse dominance order state by state and hence in expected value as well.

The reason is intuitively clear if one considers that the tariff implemented under them is ex-post efficient, whereby gross social surplus is the largest possible, and moreover the selected bidder is that with the lowest F, i.e. social surplus net of fixed costs is maximized too. For an auction rule to improve on first- and second-score ones in terms of the regulator’s welfare, it would be necessary that it pushed down the selected contractor’s expected profits. But, whatever the prior, such an auction mechanism does not exist, since to ensure incentive compatibility the contractor must always be granted the same expected profit, irrespective of the auction rules. Therefore, for the regulator there is nothing better than an auction that obtains the highest social welfare in every state, as both first- and second-score auctions actually do when bids are scored in terms of social welfare. In other words, first- and second-score auctions are optimal in expected welfare with respect to any distribution function $G(\cdot)$. All this is summarized in the following proposition.

Proposition 2. *With $m_i = m$ for all i, fixed costs i.i.d. across firms and social welfare additively separable in s, first- and second-score auctions are optimal in expected welfare with respect to any distribution function $G(\cdot)$.*

Thus with $m_i = m$ for all i second–score auctions with social welfare as score function, as well as their equivalents like first-score ones, are optimal irrespective of the prior and can be used by a non-Bayesian regulator, if he knows that F_i's are independent and that variable-cost functions do not vary significantly across firms. In other words, an ignorant and risk-neutral regulator can in these circumstances award a public service contract by scoring bids according to his welfare function $\hat{W}(\cdot)$ either in a second-score or a first-score auction (or for that matters any other auction where the most efficient bidder wins and an ex-post efficient allocation is implemented).

So far we have worked with welfare functions that do not depend on the state of the world. Now let us go back to (1). The previous results directly apply to it for $\alpha = 0$ and ensure that when $m_i = m$ for all i, to score bids the regulator should simply use the net consumer surplus

$$\hat{W}(p,s) = CS(p) - s. \tag{2}$$

But what if social welfare also depends on firms' profits besides consumer surplus, i.e. $\alpha > 0$? As a matter of fact the answer does not change: net consumer surplus remains optimal for scoring bids. To see this, suppose for a moment that the regulator knows the state of the world $(x_i)_{i \in I}, x_i = (m, F_i)$, with certainty. Since nothing in Lemma 3 depends on whether welfare is or is not affected by the state of the world, the lemma holds for welfare functions $(p,s) \mapsto W(p,s,m,F_i)$ too. In other words, by the argument of Lemma 3, we can rule out the existence of a $V(\cdot)$ supporting better outcomes than those obtained by employing $W(\cdot)$ as score function. Of course, the problem is that the regulator actually ignores the state of the world and $W(\cdot)$ cannot be used in practice. However, net consumer surplus (2) is equivalent to (1), since the second-score auction outcome is the same under both of them in every state of the world. Therefore, net consumer surplus is optimal among all score functions even with $\alpha > 0$. The intuition is straightforward. In second-score auctions the winning firm's profit is always equal to $F_2 - F_1$ irrespective of the equilibrium tariff, i.e. a change in the score function can only affect the implemented tariff and the consumer surplus. Then, if the latter has a positive impact on welfare, however small, a score function that induces the highest possible consumer surplus is certainly a solution to the problem. From Lemma 3 we know that to obtain the highest consumer surplus the trick is to use it to score bids: therefore, even when social welfare is specified as (1), it is in fact enough to adopt (2) as score function (for a formal proof see the appendix).

As we have seen the equality of variable costs across firms plays a critical role in the choice of the auction mechanism by an ignorant regulator. It is then natural at this point to ask how likely such a situation is in regulatory practice. To this purpose take as an example a service contract for the operation of bus lines. Such contracts usually impose on the contractor the duty to drive given routes with given frequency, i.e. to total a fixed amount of kilometres per time unit. In these circumstances the marginal service cost – that here coincides with the cost of an extra passenger/kilometre - is unlikely to differ significantly across potential contractors, since it essentially reflects variations in fuel consumption and tyre wear which can hardly differ among them. By contrast, fixed costs depend on the efficiency of the firm's overall organization which can indeed vary from firm to firm. Therefore cost differentials among potential contractors, if any, are in this case to be ascribed to fixed costs only. Similar remarks hold for other services of similar nature, like e.g. waste disposal, but may hold for entirely different ones too. Let us consider by way of example a contract for building and exploiting, say, a bridge (or a motorway, a tunnel, etc.). Construction entails large fixed (sunk) costs that can vary considerably across potential contractors according to their efficiency. Operating costs are mostly fixed (non-sunk) too: the bridge requires the employment of so many surveillance staff, the light of so many lamp columns, etc., irrespective of how many vehicles cross it, unless it is kept closed (in which case their cost is zero). Maintenance costs are instead partially variable with use (like e.g. the cost of asphalt paving) but the variable part is again unlikely to differ significantly across potential contractors, who often contract out maintenance and usually have access to the same contractors.

The few examples seen suggest that the cost structure under consideration is potentially relevant to many kinds of public services. Therefore the auction rules we have derived are not only interesting from a theoretical standpoint - in that they provide an example of solvable franchise-auction design problem under regulator ignorance - but also for auction practice. Finally, it is to be noted that these rules may be applicable even in situations where marginal costs do vary across firms but variations are small: in such circumstances the above auction rules are likely to be approximately optimal and therefore their application field may be in fact wider than that identified by the previous results.

Reasonable auction design

In the previous section we identified informational conditions under which a non-Bayesian regulator can solve the auction design problem in a prior-independent way, in particular:

1) *equal variable-cost functions across firms*, under which a prior-independent (pointwise) optimal score function exists in the set of second-score auctions which coincides with the social welfare function,
2) *equal variable-cost functions across firms, additively separable welfare functions and i.i.d. fixed costs*, under which first-score and second-score auctions with the regulator's objective as score function are optimal in expected value for all possible priors and among all auction mechanisms,

We have also seen that in the absence of any information (i.e. optimization over the universal domain of priors) the design problem has no solution with respect to the prior-independent optimality criterion. The absence of any information about costs, however, is not the only possible cause for the regulator's inability to make a fully rational choice: in both cases 1 and 2 the solution requires that the regulator scores bids by his natural objective which then must be known to him, but unfortunately this is not always the case. Our task in this section is to discuss how the regulator can cope with the lack of this information in situations that structurally admit solutions to the prior-independent optimization problem, i.e. such that $m_i = m$ for all i (see Proposition 1 above).

For an ignorant regulator the computation of (1) presents two problems regarding respectively cost parameters m , F , and the demand function. As we saw in the previous section, the former is easily solved if variable costs are equal across firms, because (2) can be employed as score function in place of (1) without welfare losses: full rationality can thus be preserved despite the absence of the regulator's prior on costs. But the second problem appears insurmountable.

A choice is rational if it is optimal relative to the decision-maker's objective. Here the choice concerns mechanisms that are essentially identified by a score function. If the regulator knows that variable costs do not vary across firms (and we have seen that this may be indeed the case with many public services), the rational choice is his natural objective itself, i.e. it is optimal for the regulator to use his welfare function to score bids. With demand ignorance, however, the natural objective is not computable nor is its substitute (2): the regulator is then unable to make a fully rational choice. Even more disturbing is the fact that, when he chooses any computable score function $y(p,s) \equiv v(p) - s$ with the usual properties (additively separable, continuous and monotonic decreasing) for use in a standard auction (first or second score, for example), he actually behaves as if he pursued an objective different from the true one. Indeed, every result we established in section 3 for social welfare functions also holds for generic functions $(p,s) \mapsto y(p,s)$ with the same properties (in Propositions 1 and 2 just replace $W(\cdot)$ with $y(\cdot)$). In particular, by Propositions 1 and 2 the choice of a score function $y(\cdot)$ under our conditions "maximizes" the score function itself, i.e. induces the best outcome with respect to $y(\cdot)$ itself and this will in general not be a social optimum. As a matter of fact, in this way the regulator may occasionally attain a social optimum but he can never know whether this is the case and, if it is not, how big the welfare losses associated to different scoring rules are: what he knows is just that on choosing a certain score function in fact he sets it as his maximand. To stress that the operational objective $y(\cdot)$ pursued in practice by the regulator is a second objective that coexists with the natural one – the social welfare function – we call it *pseudo-objective*.

Under our conditions to maximize a pseudo-objective one has to employ it as score function; viceversa, the choice of any computable score function optimizes a pseudo-objective that coincides with it. Given this one-to-one correspondence, evaluating a score function here amounts to evaluating the pseudo-objective to be maximized. In view of this we can say that the choice of a score function is *reasonable* if a reasonable pseudo-objective is chosen. Note that at this point the analysis focus shifts from the plane of mechanisms to that of objectives: in other words, the regulator's question is no longer "by what instruments can I pursue my objective?", as in standard mechanism theory, but "which objective am I to pursue?". Of course, the real problem is to define what a reasonable pseudo-object is - a problem that can have many solutions, both in theory and in practice. This issue falls outside the scope of the present paper but in the remaining part we want to discuss a few seemingly indisputable requirements of reasonableness to show that even the application of minimal requirements can prove useful for the choice of a score function in practice.

A first obvious requirement is that they be computable. The previous discussion about the non-implementability of social welfare in the presence of demand ignorance centred around computability. Since the pseudo-objective is to be used as score function, then it is out of question that non-computable ones can be regarded as reasonable choices at all.

Moreover, franchise auctions are instruments for selecting contractual terms like tariffs and subsidies which are economic variables. Then it seems also sensible to require that the regulator optimize some economic evaluation of these variables. Indeed, without any restriction in this sense choices that essentially concern economic matters would be completely arbitrary from an economic standpoint. In other words, in matters like the supply of public services which entail cost and benefits to citizens it seems reasonable that the choice made maximizes some benefit or minimizes some cost or optimizes an appropriate mix of the two. One can debate which costs and benefits are to be maximized/minimized in each situation but it is hard to see how a reasonable choice can avoid to account explicitly for them. In a way, by imposing this requirement one compels the regulator to be explicit about the costs or benefits (or both) of his choices.

To sum up, for pseudo-objectives/score functions to be reasonable it seems sensible to require that they

R1) be computable,

R2) evaluate economic bid variables according to some clearly defined notion of economic cost or benefit.

How these requirements can be specified and used in practice is illustrated next by a few examples. In the following subsections we discuss two pseudo-objectives/score functions in the light of the minimal criteria R1 and R2. The first of them is not known to be in use and is proposed here as a possible solution to the auction design problem for a practically relevant class of situations. The other is drawn from experience and is an example of unreasonable score function by the standards we have set.

Minimize maximum cost of service

Service contracts sometimes set a rigid capacity constraint on supply. A clear example is parking services produced on a given parking lot: the number of parking hours “produced” per day is equal to the number of parking places times the daily opening hours and not a single hour more can be made available by the given facility (it is irrelevant here if the parking facility already exists or must be built under the contract: a BOT contract which sets a given size for the parking lot to be built presents the same problems as contracts for the operation of an existing one). The maximum capacity $\bar{y}$ of course coincides with the fixed service supply and all costs are in fact fixed ($c(\bar{y}, m)$ is an uncertain constant too). In these circumstances the first-best tariff is $P(\bar{y})$ as in Fig. 4 ($P(\cdot)$ is the inverse of the demand function $y(\cdot)$).

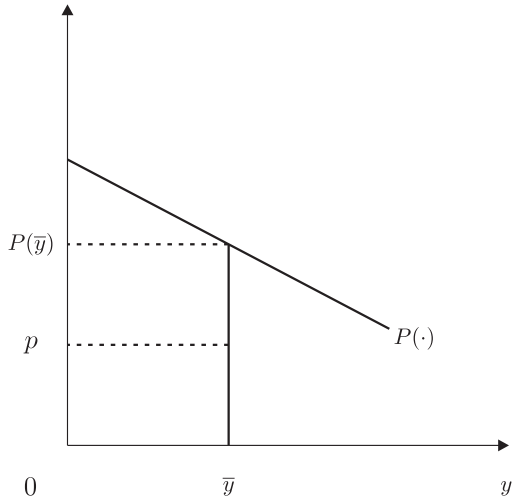

Figure 4

When a rigid production capacity is set by the contract to be awarded, this information is common knowledge and can be exploited to define an acceptable score function/pseudo-objective even by an ignorant regulator. Take for example the sum of subsidy and tariff times maximum capacity, i.e.

$$K(p,s) = p\overline{y} + s \,. \tag{3}$$

This represents the *maximum cost* that citizens commit themselves to bear for a maximum capacity equal to $\overline{y}$ when the contract awarded through the auction is (p,s). Since scores $K(p,s)$ are computable for every possible bid and are measures of a definite cost notion, both requirements R1 and R2 are met and hence to minimize this function can be regarded as a reasonable objective for the regulator (with respect to social welfare it is obviously better to minimize than maximize it). Indeed, the level of service supply $\overline{y}$ chosen before calling for competition is that judged adequate to citizens' needs. Therefore, in the absence of any information on service use, it does not seem unlikely that citizens would consider acceptable to minimize the maximum cost of service supply. Minimizing pseudo-objective $K(\cdot)$ is equivalent to maximizing $-K(\cdot)$ and the latter is monotonic decreasing in both variables, continuous and additively separable. Then, if $m_i = m$ for all i, Lemma 3 and Proposition 2 apply to this function and, in order to minimize the expected value of maximum service cost, the regulator has just to run a first- or second-score auction and award it to the bidder whose bid solves

$$\min_i (p_i \bar{y} + s_i) .$$

Remark

The standard second-score rule presents a minor problem with score functions (3). For $p \leq P(\bar{y})$ isoprofit and isoscore curves are line segments with the same slope $-\bar{y}$ and therefore have an infinity of tangency points. This implies that under the second-score rule the winner might end up choosing a contract with a tariff strictly smaller than $P(\bar{y})$, that is a suboptimal tariff in terms of social welfare. Despite the fact that the regulator does not know the social welfare function, he can in this case implement the first-best tariff by a slightly modified awarding rule: among all bids with the same score there wins that with the highest tariff and the implemented tariff is the winner's. Every bidder has an incentive to quote $P(\bar{y})$ and the implemented subsidy is the second-highest one (all bids on the linear tract of the isoprofit curve are indifferent to firms and hence none finds advantageous to offer less than $P(\bar{y})$).

The basic objective $K(\cdot)$ can also be refined in many ways to allow for additional information the regulator may have. In order are discussed a few variants of it by way of example.

1) If the regulator wants to take account of the different outlay timing of subsidy (upfront payment) and tariff (whose payment is diluted over time), scores (3) can be adjusted by allowing for the cost of capital advances as in e.g.

$$p\bar{y} + (1 + r)s$$

where r is an appropriate interest rate that measures that cost (this formula corresponds to payment of subsidy in this period and payment of tariff in the next one; of course any other timing structure can be accounted for by modifying it suitably).

2) The regulator may also want to take into account the shadow cost of public funds, (subsidies are usually financed through distortionary taxes) in which case (3) can be transformed into the following

$$p\bar{y} + (1 + \lambda)s .$$

3) In the previous cases maximum cost is calculated with respect to maximum capacity. Sometimes, however, it is a priori known that this will never be used in full. An example is again offered

by parking services. If a parking lot is open 24 hours, its maximum capacity is 24 hours times the number of parking places. However, because of demand variability over the day, parking lots are usually designed to satisfy a quota of the foreseeable peak-time demand, though this implies overcapacity off peak-time (e.g. at night). On the basis of experience the regulator may be able to identify an upper bound for service use that is independent of the tariff and lower than $\bar{y}$ (a parking lot in a commercial area will never be full at night even if free). Then, if it is known by experience that total use will in no case exceed a portion $b < 1$ of supply, the evaluation of the maximum service cost can be made more precise by disregarding the part of total supply that is never used and hence it may be reasonable to score bids according to

$$b p \bar{y} + s \,.$$

1.1 Maximize average percent decrease relative to reserve values

The regulator's ignorance, as we have said, prevents him from making a strategic use of reserve values, which will thus be determined previously to the choice of auction rules (e.g. by taking the maximum of each variable he is willing to accept). Sometimes reserve values, however chosen, are used to score bids and not only to exclude "bad" ones. One method used in practice is to rank bids by the realized average percent decrease relative to reserve values.[1] Let x_j be a generic bid variable whose maximum reserve value is $\bar{x}_j <$ ¥ . Firm i's bid on this variable, x_{ji}, receives a sub-score

$$\frac{(\bar{x}_j - x_{ji})}{\bar{x}_j}$$

The total score made by i is then obtained as weighted sum of these sub-scores over all j

[1] For example it is found in Tuscany's regional statute Regolamento Regione Toscana n. 514, 26/4/96.

$$\sum_j g_j \frac{(\bar{x}_j - x_{ji})}{\bar{x}_j} \qquad (4)$$

where g_j are predetermined constants. Since $[(\bar{x}_j - x_{ji})/\bar{x}_j]$ represents the percent decrease of bid x_{ji} relative to the reserve value $\bar{x}_j$, then (4) is the weighted average of the cuts offered by bidders on each variable. The question is if it is reasonable to use this method in our context where the concerned variables are tariff and subsidy, i.e. to use the score function

$$g_1 \left(\frac{\bar{p} - p_i}{\bar{p}} \right) + g_2 \left(\frac{\bar{s} - s_i}{\bar{s}} \right). \qquad (5)$$

At first sight this criterion may appear acceptable, or at least as acceptable as the previous ones, but at a closer examination it is not so. Under the conditions of Proposition 1, choosing a bid with the highest value of (5) amounts to "maximizing" the average percent decrease proposed by bidders relative to the maximum that citizens are willing to pay for the service supply either in the form of subsidy or tariff: in other words, (5) is in our context the best score function if the regulator's pseudo-objective is to attain the highest percent average decrease relative to reserve values. Does this really make sense? A given percent cut in the tariff, which is an average cost, is here valued equivalent to g_2 / g_1 times the same cut in subsidy, that is a component of total cost. Then we have a comparison between two heterogeneous and incomparable magnitudes (subsidy could be weighed against total tariff expenditure, not against the unit tariff). This negative conclusion cannot be reverted by arguing that the product of tariff times coefficient g_2 / g_1 is a measure of tariff expenditure: in general it is not so because the latter is not a linear function of the tariff (such a function is indeed completely unknown to the regulator). Therefore by adopting (5) the regulator "maximizes" something that can in no way be regarded as an economic evaluation of bid variables: the second reasonableness requirement R2 is thus violated and the criterion is to be rejected as unreasonable.

As the second example has shown, weak though they are, the above minimal reasonableness requirements can be effective in practice for restricting the choice field. Of course, after depuration through them is made, there will usually remain more than one reasonable score function to choose from. It would then be important that the regulator explicitly stated what he is trying to attain with the choice. We have seen however that this is not always possible since under the severe informational constraints often observed in practice no definite outcomes may be associated to his choices: our analysis has made clear under what technological conditions this can actually be done.

CONCLUSIONS

One of the most critical problems of Bayesian auction models is that they postulate an amount of a priori information that is usually unavailable to real-world auctioneers. As a consequence, the optimal auction rules devised by Bayesian theory turn out inapplicable in many practical situations. This paper has made a first step towards bridging the gap between theory and practice with reference to a concrete problem - the contracting out of public services. We have focused on a regulator endowed with very little information about technology and market conditions and found that, even under the most severe informational constraints, in certain circumstances he can design auctions for public service contracts that, if not fully rational, are at least reasonable: when production costs have a certain structure (namely, cost variations among firms are mainly imputable to differences in fixed costs), the question of how to structure franchise auctions gets precise answers and, above all, answers that can be immediately applied in practice. A limit of these results is that, though relevant for many public services, they do not hold outside the specific conditions identified here. However, when marginal costs vary across firms and their variations are small, the auction rules we have devised are likely to be approximately optimal. That is, by adopting a suitable notion of approximate rationality/reasonableness we are likely to get a larger class of environments to which our rules are applicable - a task we leave to future research.

APPENDIX

Proof of Lemma 2

Define couple b as

$$b \equiv \arg\max_{p_2 = 0} \hat{W}(p,s)$$

and a as

$$a \equiv \arg\max_{W(p,s) \geq W_2^*} p_1(p,s)$$

for $\hat{W}_2^* = \hat{W}(b)$.

If in all states either $\pi_1(a) = 0$ the social welfare function $\hat{W}(\cdot)$ supports the full-information optimum in each state irrespective of whether $a = b$ or $a \neq b$ and then there exists no score function that allows to obtain better outcomes to the regulator than score function $\hat{W}(\cdot)$. Let us then focus on the case where in at least one state there simultaneously hold $\pi_1(a) > 0$ and $a \neq b$, i.e. $p_a \neq p_b$, since $\hat{W}$ is strictly decreasing in both variables.

As is immediate to verify, in the given circumstances there always exists a point $c \in [0,\infty[\times \mathbb{R}$ such that

$$\pi_1(c) \geq 0,$$

$$\hat{W}(c) > \hat{W}_2^*.$$

To see this it is enough to take $c = a + (0,-\delta) = (p_a, s_a) + (0,-\delta), \delta > 0$. Since $\pi_1(p_a, s_a - \delta) = \pi_1(a) - \delta$, we can take for example $\delta = \pi_1(a)/2$. Strict monotonicity of $\hat{W}$ ensures $\hat{W}(c) > \hat{W}_2^*$.

Now denote the operational profits $\pi_O^i(p) \equiv py(p) - c(y(p), m_i) - F_i$ $(\pi_O^i(p,s) = \varphi_i(p) + s)$ and consider the following functions of p

$$p \mapsto \pi_O^1(p) + k_1$$

$$p \mapsto \pi_O^2(p) + k_2$$

where k_1 and k_2 are chosen so that $\pi_O^1(p_a) + k_1 = \pi_1(a) - \delta$ and $\pi_O^2(p_d) + k_2 = 0$. Given the continuity of the two functions, the minimum function $p \mapsto \min\{\pi_O^1(p) + k_1, \pi_O^2(p) + k_2\} \equiv h(p)$ is continuous too. Further take a function $\mu : [0,\infty[\to \mathbb{R}$ such that

$$\mu(p_c) = 0$$

$$\mu(p_d) = 0$$

$$\mu(x) < 0, \forall x \geq 0$$

(functions with these characteristics and even of class $C^\infty[0,\infty[$ can be defined without difficulty). Finally define $V(p,s) \equiv h(p) + \mu(p) - s$.

Function V is continuous and has value 0 if and only if $h(p) + m(p) = s$. Moreover, $V(p,h(p)) \equiv m(p) \leq 0$ (namely, it is always < 0 except at p_c and p_d). Therefore points c and d function V are maximizers of V since

$$V(p,s) \equiv h(p) + m(p) - s \leq h(p) + m(p) - h(p) = m(p) \leq 0$$

for $s \geq h(p)$. We have thus proved the existence of an iso-score curve like V in Fig. 1. Then by translating the curve vertically we get a score function $V : [0,\infty[\times \mathbb{R} \to \mathbb{R}$ that dominates $\hat{W}$ in the state of the world under consideration and therefore we can conclude that in the given conditions $\hat{W}$ is not pointwise optimal.

Q.E.D.

Proof of Proposition 1

(Sufficiency) The sufficiency part immediately follows from Lemma 3.

(Necessity) Given any m_1 and m_2 such that $m_1 \neq m_2$, there exists a state of the world where bidder 1 is the winner and bidder 2 is ranked second: to find it simply set 2's fixed cost high enough to be second and all others' costs high enough to be third or lower. Then if m_i are allowed to differ across firms, there is always a state of the world where the highest and the second-highest bidder have different variable-cost parameters.

It is immediate to realize that second-score auctions under $\hat{W}(\cdot)$ cannot support a full-information optimum in every state of the world in which $m_1 \neq m_2$. A full-information optimum is implemented in some state only if there is a tie between the first and the second bid, i.e. the score obtained by them is the same

$$\hat{W}(p_1^*, s_1^*) = \hat{W}(p_2^*, s_2^*)$$

where

$$(p_i^*, s_i^*) \equiv \arg\max_{p(p,s,m_i,F_i)=0} \hat{W}(p,s) \qquad (6)$$

In such a case the contract implemented by 1 is just (p_1^*, s_1^*): since $\hat{W}(p_1^*, s_1^*) = \hat{W}(p_2^*, s_2^*)$ is the highest score subject to the zero-profit constraint, by duality there is not attainable a higher profit subject to the constraint $\hat{W}(p_1, s_1) = \hat{W}(p_2^*, s_2^*)$. Of course for fixed costs

$F_1 - \varepsilon \geq 0, \varepsilon > 0$ there must be $\pi(p_1^*, s_1^* - \varepsilon, m_1, F_1 - \varepsilon) = 0$, and $\hat{W}(p_1^*, s_1^* - \varepsilon) > \hat{W}(p_2^*, s_2^*)$, i.e. in that state there is no tie. We have thus proved that for any $\hat{W}(\cdot)$ there are states of the world with $m_1 \neq m_2$ and where the second-score auction does not implement a full-information optimum under it. Since the single-crossing property implies that in no state of the world the implemented contract coincides with the second bid, Lemma 2 applies to every state $\{(m_i, F_i)\}_{i \in I}$ such that $m_1 \neq m_2$. Then we can conclude that $\hat{W}(\cdot)$ is not pointwise optimal if m_i are not equal across firms in all states of the world.

Q.E.D.

Proof of Lemma 4

The proof follows the standard differential method by Milgrom (1985, 1989) (here all functions are assumed differentiable). Since m is common knowledge to firms, the uncertain variables to firm i are $(F_1, \ldots, F_{i-1}, F_{i+1}, \ldots, F_n)$. Variable m is a parameter of firms' optimality calculations and therefore we can represent the problem as if we had has as many auctions as the values of m. The optimal expected profit for a generic firm of type F, given the optimal winning probability $\varphi^*(m,F)$, is

$$\pi^*(m,F) = \varphi^*(m,F)\left[p^*(m,F)y(p^*(m,F)) + s^*(m,F) - c(y(p^*(m,F)),m) - F\right]$$

By the Envelope Theorem we have

$$\pi^{*\prime}(m,F) = -\varphi^*(m,F)$$

from which by integration there follows

$$\pi^*(m,F) = \int_F^{\bar{F}} \varphi^*(m,z)\,dz$$

($\pi^*(m,\bar{F}) = 0$: the probability of winning for the worst type $\bar{F}$ is null and so is the optimal expected profit in $\bar{F}$). Therefore a generic bidder's expected profit conditional on m is:

$$E_F(p^*(m,F)) = \int_{\underline{F}}^{\bar{F}} \int_{F}^{\bar{F}} f^*(m,z) g(F) dF dz$$

The winner's expected profit (i.e. society's expected outlay) is tehrefore $nE_F(p^*(m,F))$. In all auctions awarded to the lowest F (given m) the winning probability $f^*(m,F) = [1 - G(F)]^{n-1}$ is the same and therefore the same is the expected profit conditional on m. The conventional surplus, defined as $[v(p) + py(p) - c(y,m) - F]$, is in expected value the same in all auctions that select the lowest-F firm and entail the same tariff outcome (that is if $p^*(m,F)$ is invariant to the auction rules), i.e.

$$E_{F_{(1)}} \{v(p^*(m,F_{(1)})) - s^*(m,F_{(1)}) + [p^*(m,F_{(1)}) y(p^*(m,F_{(1)})) + s^*(m,F_{(1)}) - c(y(p^*(m,F_{(\ldots$$

where $F_{(1)}$ is the first-order statistics of F. The sum in square brackets is the winner's expected profit, which is equal in all auctions under consideration. Then, by difference, both the expected score conditional on m, $E_{F_{(1)}}[v(p^*(m,F_{(1)})) - s^*(m,F_{(1)})]$, and its expected value with respect to m are equal too.

Q.E.D.

Equivalence of welfare functions derived from social surplus

First we verify that net consumer surplus is equivalent to (1) as score function, in the sense that, even if the regulator knew each bidder's parameters and were able to compute the social welfare (1) for each bid, the outcome of second-score auctions awarded in this way would be the same as net consumer surplus as score function. So for argument's sake suppose that the regulator knows the state of the world $(m_i, F_i)_{i \in I}$ at moment of evaluating bids. To prove equivalence we must ascertain that in switching from one to the other score function there do not change: a) the winner, b) the implemented contract.

It is immediate to verify that the winner does not change with the score function. Denote operational profits (i.e. net of subsidy) $\pi_O(p,m,F) \equiv py(p) - c(y(p),m) - F$ and let (p_i^*, s_i^*) be a solution to

$$\max_{p,s}\left[CS(p) - s + \alpha p_O(p,m_i,F_i) + \alpha s\right] \quad (7)$$

subject to

$$s + p_O(p,m_i,F_i) = 0$$

Since by (7)

$$CS(p_i^*) - s_i^* + \alpha p_O(p_i^*,m_i,F_i) + \alpha s_i^*) \geq CS(p_j^*) - s_j^* + \alpha p_O(p_j^*,m_j,F_j) + \alpha s_j^*)$$

implies

$$CS(p_i^*) - s_i^* \geq CS(p_j^*) - s_j^*$$

then the ordering of bidders is the same both under (2) and (1) as score functions.

Let us now turn to the second problem. Assume that under both score function the winner is 1 (identified by (m_1, F_1)) and the second-highest is 2 (m_2, F_2). The second-highest bid is solution to

$$\max_{p,s}\left[CS(p) - s + \alpha p_O(p,m_2,F_2) + \alpha s\right]$$

sub:

$$s + p_O(p,m_2,F_2) = 0$$

Firm 2's tariff is clearly also solution to

$$\max_{p}\left[CS(p) + p_O(p,m_2,F_2)\right]$$

i.e. it is that maximizing social surplus irrespective of α. Denote W_2^* the social welfare associated to the second-highest bid. The tariff implemented in equilibrium is solution

$$\max_{p,s} \pi(p,s,m_1,F_1) \quad (9)$$

subject to $CS(p) - s + \alpha(p_O(p,m_1,F_1) + s) = W_2^*$

i.e. it is solution of

$$\max_{p}\left[p_O(p,m_1,F_1) + \frac{1}{1-\alpha}\left(CS(p) + \alpha p_O(p,m_1,F_1) - W_2^*\right)\right]$$

which we can also write as

$$\max_p \left\{ \frac{1}{1-\alpha}\left[CS(p)+\pi_O(p,m_1,F_1)\right]-\frac{W_2^*}{1-\alpha} \right\}.$$

Therefore the outcome tariff $p^*(m_1)$ does not vary with parameter α and so does the outcome subsidy $s^*(m_1,F_1)$. In other words, it is the same that would obtain by applying the consumer surplus as score function, i.e. by solving (8) and (9) for $\alpha = 0$.

The last task is to check that when we use $(p,s) \to CS(p) - s$ to score bids there exists no other $(p,s) \to V(p,s)$ that allows to attain a better outcome with respect to social welfare $CS(p) - s + \alpha\pi(p,s,m,F)$ in at least one state. If the regulator knew the state of the world, he could use $W(p,s,m,F) \equiv CS(p) - s + \alpha\pi(p,s,m,F)$ as score function. Since both welfare and profit functions are vertical translations when $m_i = m$ for all i the implemented contract and the second bid coincide and the argument of Lemma 3 can be applied state by state to this function too. This means that there exists no score function $V(\cdot)$ under which the auction outcome obtains a strictly higher welfare $CS(p) - s + \alpha\pi(p,s,m,F)$. But, since net consumer surplus $CS(p) - s$ allows to attain the same outcomes as $W(\cdot)$ in every state, there exists no $V(\cdot)$ that strictly dominate the former in terms of welfare $CS(p) - s + \alpha\pi(p,s,m,F)$. In conclusion, by resorting to (2) there is no loss due to the informational constraint that makes (1) actually unusable for scoring bids.

REFERENCES

Armstrong M., "Multiproduct Nonlinear Pricing", *Econometrica*, 64 (1996), 1-76.

Armstrong M., Rochet J.-C., "Multi-Dimensional Screening: A User's Guide", *European Economic Review*; 43 (1999), 959-79.

Asker J., Cantillon E., "Properties of Scoring Auctions", mimeo, Harvard University, 2004.

Baron D.P., Myerson R.B., "Regulating a Monopolist with Unknown Costs", *Econometrica*, 50 (1982), 911-930.

Branco F., “Auctioning Incentive Contracts: The Common Cost, Independent Types Case”, *Journal of Regulatory Economics*, 7 (1995), 277-292.

Branco F., "The Design of Multidimensional Auctions", *Rand Journal of Economics*, 28 (1997), 63-81.

Bushnell J., Oren S., “Internal Auctions for Efficient Sourcing of Intermediate Products”, *Journal of Operations Management*, 12 (1995), 311-20.

Che Y.-K., "Designing Competition through Multidimensional Auctions", *Rand Journal of Economics*, 24 (1993), 668-680.

Hansen R.G., "Auctions with Endogenous Quantity", *Rand Journal of Economics*, 19 (1988), 44-58.

Jehiel P., Moldovanu B., Stacchetti E., "Multidimensional Mechanism Design for Auctions with Externalities", *Journal of Economic Theory*, 85 (1999), 258-93.

Laffont J.J., Maskin E., Rochet J.-C., “Optimal Nonlinear Pricing with Two-Dimensional Characteristics”, in T. Groves, R. Radner, S. Reiter (eds.), *Information, Incentives and Economic Mechanisms*, Minneapolis: University of Minnesota Press, 1987, 256-266.

Laffont J.-J., Tirole J., "Auctioning Incentive Contracts", *Journal of Political Economy*, 95 (1987), 921-937.

Laffont J.-J., Tirole J., *A Theory of Incentives in Procurement and Regulation*, Cambridge (Mass.), MIT Press, 1993.

McAfee R.P., McMillan J., "Competition for Agency Contracts", *Rand Journal of Economics*, 18 (1987), 296-307.

McAfee R.P., McMillan J., “Multidimensional Incentive Compatibility and Mechanism Design”, *Journal of Economic Theory*; 46 (1988), 335-54.

Milgrom P., "Auctions and Bidding: A Primer", *Journal of Economic Perspectives*, 3 (1989), 3-22.

Milgrom P., "The Economics of Competitive Bidding: A Selective Survey", in L. Hurwicz, D. Schmeidler, H. Sonnenschein (eds.), *Social Goals and Social Organizations*, Cambridge: Cambridge University Press, 1985, 261-289.

Riordan M., Sappington D., "Awarding Monopoly Franchises", *American Economic Review*, 77 (1987), 375-387.

Rochet J.-C., “Monopoly Regulation with Two-Dimensional Uncertainty”, mimeo, Université Paris IX Dauphine, Paris, 1984.

Rochet J.-C., Choné P., “Ironing, Sweeping, and Multidimensional Screening", *Econometrica*, 66 (1998), 783-826.

Rochet J.-C., Stole L.A., “Nonlinear Pricing with Random Participation”, *Review of Economic Studies*, 69 (2002), 277-311.