AN ECONOMIC APPROACH TO PUBLIC PROCUREMENT

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ABSTRACT. Award systems play the central role in public procurement, since they determine what is considered by the contracting authority as 'the most economically advantageous tender.' Many award systems that are used in practice have serious shortcomings, which are caused by the use of relative scores. In this article, the consequences of those shortcomings are demonstrated, using examples from real procurement procedures and case law. The examples are analyzed with methods from econometrics, social choice theory and game theory.

INTRODUCTION

When a procedure for the award of a contract is initiated, the criteria are drawn up to express the contracting authority's precise needs in a formal manner. From the supplier's point of view this is self-evident, since the decision as to whether or not his proposal will be the winner should be solely based on the criteria. If the award criterion is 'the lowest price', there is not very much to say about a tenderer's winning strategy. Throughout this paper, therefore, the criterion of 'the most economically advantageous tender' is presumed. Clearly, any strategy can only be developed by the tenderers if details of the award system, such as the weighting factors and the formulas used, are published in the contract documents. If an award system is fully transparent, the tenderers will be able to calculate their scores and thus optimize their tender while they are drafting it. In practice most award systems use relative scores, which imply that calculating one's score is impossible without having knowledge of the other tenders. Under those circumstances

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optimizing a tender can only be performed using game theoretic methods. In the case law of the European Court of Justice some indications can be found to argue that under European procurement law relative scores should not be used and fully transparent award systems should be used which allow an economic optimization.

THE RANKING PARADOX

Award systems quite often have a hidden flaw, which may lead to surprising results. A simple example is the following system where there are two criteria, price and quality, with equal relative weighting and each having possible scores between 0 and 50 points. Suppose that the score for price is determined by giving the tender with the lowest price 50 points, the next lowest price 45 points, the lowest but two 40 points and so on. Such systems based on the ranking of the prices are sometimes used by contracting authorities in The Netherlands. The procedure might result in Table 1.

TABLE1
Possible Result of a Procurement

Tenderer	Quality Score	Price	Price Score	Total score
A	30	€1000	50	80
B	35	€1050	45	80
C	38	€1060	40	78

At first sight, there's nothing strange about these results. But what if tenderer A claims to have won, for example by showing tenderer B's proposal to be non-compliant with the contract documents? Once B's proposal is set aside, the score of tenderer C on the price criterion must in all fairness be adjusted to 45, since without B, C's tender presents the second best price. Assuming that the scores for quality are unaffected by eliminating B, with 83 points C would be the winner! In this article, this phenomenon is called a *ranking paradox*. It can easily be seen that a ranking paradox is always possible when in the award system relative scores are used, i.e., when the score of a tender for some criterion is determined by comparison with other tenders.

The Ranking Paradox with Price Formulas

When relative scores are used in price formulas, it can be calculated with mathematical certainty whether or not a ranking paradox is possible. In many formulas that are used in practice, this is the case. A widely used formula looks like:

Score = 50 × L / P
Where:
L = lowest price among the tenders, and
P = price of the tender for which the score is calculated.

The number 50 (points) is the maximum score that can be obtained for the price criterion. This formula is often used when for the other criteria a certain number of points are given and both numbers are added. For example, if the maximum score for the other criteria equals 50 points, the maximum total score will be 100 points.

If we have three tenders, for example, A, B and C with prices of €40, €50 and €80 respectively, the scores will be 50 points; 50 × 40 / 50 = 40 points; and 50 × 40 / 80 = 25 points. The difference between the scores of B and C is therefore 40 – 25 = 15 points. If, however, tender A is declared invalid afterwards, the lowest price is raised to €50 and now the score of B will be 50 points, and the score of C will be 50 × 50 / 80 = 31.25 points. So the difference between the scores of B and C has grown from 15 points to 50 – 31.25 = 18.75 points and it would be possible that the ranking between B and C is changed by declaring tender A invalid.

Sometimes the price formula is not mentioned explicitly, but if the score is calculated by comparison with the lowest price (or the average price, or the highest price); a ranking paradox probably will be possible. Let us take a look at the following award system that has been used in a procurement procedure for a very large contract. Three award criteria were used:

1. Functional solution (25%), i.e. the quality of the proposal on aspects like effectiveness, reliability and security.

2. Time for operational start (20%), i.e. a higher score will be given if a start date of the service will be proposed that is earlier than the date mentioned in the requirements.

3. Price (55%). Assuming that the lowest cost bid will receive 140 points, the other bids will receive fewer points, relative to the

price difference calculated as a percentage. For example, a bidder offering a price that is higher by 10% will receive around 127 points and a bidder offering a price that is higher by 50% will receive around 93 points.

Observing that 140/1.10 = 127.3 and 140/1.50 = 93.3, we may assume the following formula to have been used in calculating the score for price:

Score = 140 × L / P

This is basically the same formula as the one mentioned above, so with this formula a ranking paradox can occur in the same way.

A Logarithmic Price Formula

If a contracting authority wants to avoid the ranking paradox and still use a formula where the score is calculated by comparison with the lowest price, following is a formula that can be used:

Score = 100 – 50 × log (P / L) / log 2

With this formula, the tender offering the lowest price L will receive 100 points and a tender with a price P that is 2 times higher will receive 50 points. Of course the numbers 100, 50 and 2 can be adapted to fit the expected prices and the desired weighting of price. This formula was published in (Chen, 2005) and is nowadays used by a large contracting authority in The Netherlands. The ranking paradox is impossible with this formula because the difference between the scores of two tenders depends only on the ratio of the two prices and this ratio is not affected if another tender with the lowest price is declared invalid.

A Hidden Ranking Paradox

Quite often the award system has the possibility of a ranking paradox that cannot be discovered with mathematical methods. Let us take the example of a design and build contract, where the artistic value of the design and quality (including several other criteria, such as price) of the execution plan are used for evaluation. Suppose that the scores are determined by judging the design and the execution plan in two different teams, as is customary. Usually the artistic value and the quality are judged by giving marks between 1 = very bad and 10 = excellent (people in The Netherlands are familiar with this

system as it is the system used for report marks in Dutch schools). Suppose that three tenders A, B and C are judged by the two teams and the first team gives the marks 10, 6 and 5 for the artistic value. It may happen that the second team discovers that tender A is invalid, for example, because in its execution plan the delivery date is later than required in the contract documents. It is quite likely that the marks 6 and 5 that B and C have received for artistic value are rather low because of A's excellent design. If tender A had not been submitted at all, the marks of B and C might very well have been something like 8 and 6, which would raise the score of B compared to C. Even if the first team would judge the designs of B and C all over again without comparison with A, it is impossible in the minds of the team members to 'forget' the excellent design of A. The solution for this hidden ranking paradox is to give scores for artistic value without comparison with other tenders – which will not always be easily done.

THE RISK OF ALLOWING VARIANTS

Let us return to the first example of Table 1 and see what may happen if we replace B's tender by a variant submitted by A (assuming that variants are allowed). By submitting a variant with a slightly higher price (and very poor quality) A is able to increase the distance with C and thus can win the procurement with a method that is unfair – without the variant clearly C would have won (Table 2). This kind of situation may occur when the contracting authority allows the submission of variants, and at the same time, judges variants within the same award system as regular proposals. In combination with a system that uses the ranking of the prices as a basis, this is particularly risky. In Dutch case law there has recently been an example of a procurement where 9 compliant tenders had been submitted and 10 variants – the tenderer with the lowest price had

TABLE 2
Possible Consequence of a Variant

Tenderer	Quality Score	Price	Price Score	Total score
A	30	€1000	50	80
Variant of A	15	€1001	45	60
C	38	€1060	40	78

submitted 3 variants.[1] Obviously there must have been some obscurity in the award system, since otherwise it wouldn't have made sense to submit more than one variant.

Are Variants Possible in a Transparent Award System?

One might argue that the system of the European Procurement Directives contains a hidden tension between the principle of transparency and the possibility of the submission of variants. An award system can only be fully transparent if the scores of a tender and a variant can be calculated beforehand, without having knowledge of the other tenders. If relative scores are given, i.e. if the score of a tender is dependent on the other tenders, the system cannot be fully transparent. Now, it is generally assumed that submission of a variant is only allowed if at the same time a tender is submitted that is fully compliant with the contract documents. But in a fully transparent system it is possible for the tenderer to calculate the scores of the compliant tender and the variant beforehand, which means that it would be useless to submit a variant if its score would be worse than the score of the compliant tender (as was the case in Table 2). But if the variant has a better score than the compliant tender, one might wonder why the compliant tender had to be submitted. Apparently some kind of mechanism is used by the contracting authority to choose between the variant and the compliant tender (and other tenders and variants). And this mechanism introduces a form of obscurity in the system.

APPLICATION OF SOCIAL CHOICE THEORY

The ranking paradox can occur if the rule of *independence of irrelevant alternatives* is disregarded. This is a major rule of social choice theory, an econometric theory which analyses *choice rules*, i.e. rules by which the best option is selected from a number of alternatives, based on the individual preferences of a group of persons. The rule of *independence of irrelevant alternatives* says that the relative ranking of two alternatives A and B must not be affected by a third alternative C. It must be mentioned that in social choice theory the rule is not undisputed (Kelly, 1978). But most people would consider it to be fair that if three candidates A, B and C for the presidency of a country in a simple one person, one vote system would receive 40%, 38% and 22% of the votes, a second election is

held between candidates A and B, where the third candidate C as the *irrelevant alternative* is eliminated. In this second election it is quite well possible that B wins, which would prove that the initial preference of A over B was influenced by the third alternative C.

Arrow's Impossibility Theorem

The interesting thing about social choice theory is that it contains a theorem, called *Arrow's impossibility theorem*, which shows with mathematical certainty that choice rules that have a few very self-evident properties do not exist. For many sets of properties that look self-evident, it can be proven that choice rules fulfilling those properties do not exist (Kelly, 1978). It is easy to translate social choice theory to award systems by taking the tenders as the alternatives and the criteria as the individuals. Each criterion determines a ranking of preference between the tenders and the goal is to choose the best tender determined by all rankings. Now, according to social choice theory, an award system should have the following five properties (in the formulation of Blau, 1972):

1. Unanimity: if *each* criterion determines that proposal A is better than proposal B, then in the final ranking B may not be preferred to A.

2. Non-dictatorship: there may not be one particular criterion that will always determine the final ranking between the proposals, under all circumstances.

3. Universal Domain: for each set of proposals and thus for all possible rankings determined by the criteria, the award system must determine a winner (or a set of winners ex aequo).

4. Independence of Irrelevant Alternatives: in the final ranking, the relative ranking between two proposals A and B must not be dependent on a third proposal C.

5. No egalitarism: the award system may not be trivial, i.e. it should not always decide that all proposals have the same ranking.

Arrow's theorem says that if there are more than two tenders, there is no award system based on ranking alone that has the properties 1 to 5. These properties seem to be natural and rather minimal demands that an award system should fulfill. Only the second and fourth property may be not self-evident at first sight. If, for

example, in an award system the final ranking is always determined by the price criterion, the second property is not complied with. In Dutch case law there are some examples where the court decided that if the criterion of the most economically advantageous tender is chosen in the contract notice, the contracting authority may not apply the 'lowest price' criterion at a later stage.[2] This can also be inferred from the well known SIAC-judgment of the European Court of Justice, where the court rules that "the contracting authority must interpret the award criteria in the same way throughout the entire procedure."[3] So the second property – non-dictatorship – could be formulated in terms of procurement law as follows: if the criterion of the most economically advantageous tender is chosen, the criterion price may not be decisive regardless of the other criteria.

Inferences from Social Choice Theory

It must be noted that in social choice theory the basic idea is that alternatives are compared by an individual without looking how strongly one alternative is preferred over the other. Therefore this theory cannot be directly applied to most award systems. For example, if A, B and C are tenders with prices of €1,000, €1,001 and €1,200, usually A is only slightly preferred to B, but B is strongly preferred to C. Since this weight is usually expressed in a number, award systems add a new element to the choice rules of social choice. What we certainly can learn from social choice theory is that if we impose requirements on an award system that each considered on its own looks self-evident, it may very well be that as a whole these properties form a set for which it is mathematically impossible to design an award system that fulfills them all. In case law, usually the award system is not considered as a whole – in most cases the claimant points at certain weaknesses in the system which from time to time is considered unlawful by the court. In the first judgment mentioned in note 2, the judge decided that the award system was invalid because in this system an unreasonable result was theoretically possible, as shown in Table 3.

This result indeed looks very unfair, but clearly it is even more unlikely than unfair. Applying the same logic, a system for school exams might be called invalid if a candidate with the marks 5, 10, 10, 10, 10 and 10 would fail and on the other hand a candidate with the marks 6, 6, 6, 6, 6 and 6 would pass the exam (in the Dutch system a

TABLE 3
Possible outcome in the award system

Tender	Price	Price Score	Quality Score	Total Score
A	€7,500,000	170	0	170
B	€7,500,001	160	0	160
C	€7,500,002	150	0	150
D	€7,500,003	140	29	169

5 is "nearly sufficient," 6 is "sufficient," and 10 is "excellent"). Fortunately, another judge recently decided that the mere fact that a fictitious example in which the award system might lead to an unreasonable result in itself is insufficient to declare the procurement procedure invalid.[4]

In a legal system where case law has authority, it is possible that if all decisions about the validity of award systems are combined, a set of rules will be created where – like in Arrow's Theorem – no single award system can comply with. One cannot expect that judges will be able to foresee the consequences of their verdicts when in science it is still unknown under what exact circumstances these rules will be conflicting. Thus, judges should be very careful before they decide that a certain award system is invalid.

THE ECONOMIC ASPECT OF THE AWARD CRITERIA

In a fully transparent award system, the tenderers will be able to calculate their score when they have formulated their proposal. A very simple example of such a system is a procurement for the supply of laptops where there are two criteria used: price in euro and warranty period in months. If the expected price of one laptop is around €600, the following formula could be used.

Score = 1000 – P + W
Where
  P = price, and
  W = warranty.

In other words, the price is subtracted from 1000 after which the number of months of the warranty period is added. In this award system it would be wise for the contracting authority to define a lower

bound and an upper bound for W, for example 12 months and 36 months respectively. If not, a tenderer might offer an unrealistic value like 120 months. The formula expresses the desires of the contracting authority in a very clear way. For example, a laptop with a price of €600 and a warranty period of 12 months has the same score as a laptop with a value of €599 and a warranty period of 13 months. One might say that in this system the price of an extra month warranty equals €1. Now, if we consider combinations like (€600, 12 months), (€599, 13 months), (€598, 14 months), (€597, 15 months) and so on it is clear that all of them will lead to the same score of 1000 – 600 + 12 = 412 points. Such combinations of two real numbers can be seen as points in a two-dimensional plane. It is easily seen that the set of all points with score 412 form a curve, which is called an *indifference curve* in economics. In this simple example the indifference curves are straight lines. When the indifference curves are known, it is possible to find the optimal values for the tender, as will be shown with the next example.

Using Indifference Curves for Optimization

The following more realistic example of an award system will be used to describe how a tender can be optimized. In public procurement for the hosting of a software system one of the important criteria is the availability of the system, expressed as a percentage of the time during which the system is "live." For example, 99.5% availability means that in the course of a year, the system will not be out of operation due to unforeseen circumstances for more than 0.5% of the time. The score for price is calculated as follows:

1. 80 points are awarded for a price of €100,000;

2. 1 point is deducted for every €10,000 by which the price exceeds €100,000.

A price of €120,000 would thus earn 80 – 2 = 78 points.

The availability score is calculated as follows:

1. 10 points are awarded for 99.0% availability;

2. 1 point is added for every 0.1% by which the availability offered exceeds 99.0%.

So an availability of 99.3% would earn 10 + 3 = 13 points. This award system, although it looks a little more complicated, is basically

the same as the award system used in the example of the procurement of laptops, since it also uses only linear functions. Therefore this award system also has straight lines as indifference curves. Figure 1 gives three such straight lines. All points on the upper line indicate combinations of availability and price with a score of 86 points. For example, with availability at 99.0% and price at €140,000, the score received equals 10 + 76 = 86 points. Availability at 99.1% and price at €150,000 scores 11 + 75 = 86 points, and so on.

It must be stressed that the straight lines represent equal return *for the contracting authority*. For the tenderer, the law of diminishing marginal return means that the indifference curves will not be straight lines – every additional euro spent on improving availability has a diminishing effect. For instance, raising the availability from 99.0% to 99.1% might entail extra cost of approximately € 8,000, but raising it by another 0.1% to 99.2% would involve a larger investment. Figure 1 shows a possible indifference curve from the tenderer's perspective that is actually curved. Of course, for a particular tenderer there are several more or less 'parallel' curves, each representing a

**FIGURE 1**
Finding the Optimal Tender
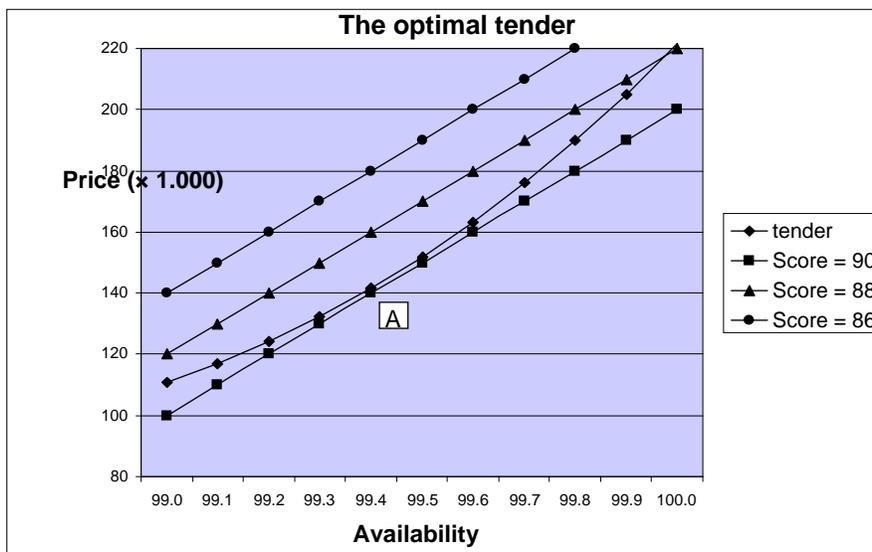
certain profit margin. However, once the desired profit margin is determined by the tenderer's internal policy, the indifference curve is fixed and the optimization can be performed.

The tenderer now has an easy task deciding on the economically most advantageous tender from his point of view – it is point A in Figure 1, providing an availability of 99.4%, a price of €140,000 and a score of 90. All other combinations on the tenderer's indifference curve yield a lower score and thus a lower utility for the contracting authority.

Optimization with Unexpected Indifference Curves

It is conceivable that the indifference curves for some tenderers are straight lines or even curves like the one shown in Figure 2. In this case the optimal tender is indicated by the capital A: an availability of 100% and a price of €200,000. Under such circumstances it is possible that the winning tender has an extremely high or extremely low value for availability. This will not be a problem if the contract documents define the lower bound and upper bound of the values that may be offered.

FIGURE 2
Determining the Optimal Tender
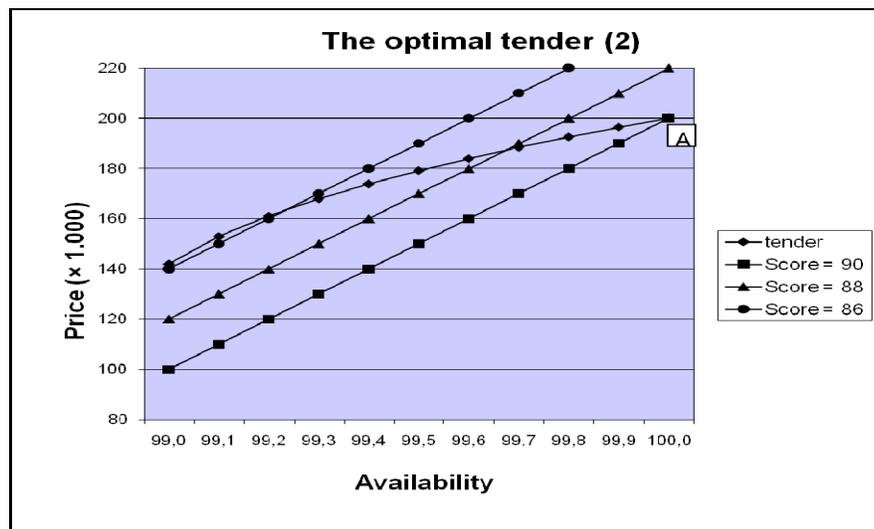
Optimization with More Than Two Criteria

If criteria additional to price and availability are imposed, the tenderer can still apply this method provided that each criterion is only related to price and not to another criterion. If another criterion is system performance, for example, then this should not be affected by availability and vice versa. In this case, it is possible first to optimize availability against price with an arbitrarily chosen fixed value for performance, followed by optimizing the combination of price and performance with availability fixed at its optimum value. The criteria are sometimes interrelated, however, in which case optimization of the values requires multidimensional, more complicated techniques. Unfortunately, many award systems do not allow for economic optimization at all.

TRANSPARANCY IN EUROPEAN CASE LAW

Clearly, if the award system is not transparent, the tenderer will not be able to choose between the optimal point and other points on his indifference curve as was possible in Figures 1 and 2 – from his/her point of view all these points are equivalent, i.e. will yield the same profit margin for him/her. It is therefore very likely that in a non-transparent award system the tender submitted is not the optimal tender. By using a transparent award system where the indifference curves are known beforehand, the contracting authority could have received a different proposal that has more "value" for the contracting authority and at the same time is equivalent for the tenderer.

The European Court of Justice has given some important judgments about the transparency that contracting authorities should observe. In the Lianakis case, the Court decides that "Article 36(2) of Directive 92/50 precludes the contracting authority in a tendering procedure from stipulating at a later date the weighting factors and sub-criteria to be applied to the award criteria referred to in the contract documents or contract notice."[5] In this judgment the Court repeats a decision from another judgment, the ATI case. In the latter case the contracting authority had made a subdivision of an award criterion into several subheadings with different weightings after the submission of the tenders, but before the opening of the envelopes. This seemed to be an act that violated the principle of transparency. But the Court decided:

Community law does not preclude a jury from attaching specific weight to the subheadings of an award criterion which are defined in advance, by dividing among those headings the points awarded for that criterion by the contracting authority when the contract documents or the contract notice were prepared, provided that that decision:

- Does not alter the criteria for the award of the contract set out in the contract documents or the contract notice;

- Does not contain elements which, if they had been known at the time the tenders were prepared, could have affected that preparation;

- Was not adopted on the basis of matters likely to give rise to discrimination against one of the tenderers.[6]

At first sight it looks as though the Court does not require a full transparency of the award system. But if we look closer at the second condition under which a specification of weightings after the submission of the tenders can be performed, the Court says that this is allowed if it wouldn't have made any difference had the tenderers known this specification beforehand. Implicitly the Court says that the tenderers have to know all details of the award system that may influence their strategy – in other words, a full transparency in the mathematical sense is required. The third condition implies that making a specification at a later stage may only be performed without having knowledge of the tenders submitted – otherwise it could be possible to let a particular tenderer win the procedure.

Relevant and Irrelevant Weightings

As we have seen before, if a tenderer has several options for his tender, he/she must know the indifference curves of the award system in order to be able to choose the optimal option for him/her. So the weighting of a criterion which leaves several options open to the tenderer is relevant. But it is possible that the weighting of (subheadings of) a criterion is irrelevant for the tenderer, simply because he/she has only one option. Suppose that, making a slight variation of the criterion of the important Wienstrom case, [7] in a procurement for the supply of electricity there are two subheadings of the criterion "ability to supply electricity from renewable energy sources:"

1. The quantity of electricity from renewable energy sources supplied in the past two years; and

2. The quantity of electricity from renewable energy sources to be supplied in the next two years.

The weighting of the first subheading is irrelevant for the preparation of the tender, since the tenderer can only give one answer. (It was argued by the claimant in the Wienstrom case that this first subheading was in fact a selection criterion and thus illegal). On the other hand, the weighting of the second subheading is very relevant, since here the tenderer will have the option to enlarge the quantity of 'green electricity' to be produced in the future. Knowing this weighting, it would be possible to calculate whether it is worthwhile to enlarge that production and adjust the price accordingly. Clearly, the Court has ruled in the ATI case that relevant weightings must be published in the contract documents.

Although the weighting of the first subheading is irrelevant in the mathematical sense, it still can be very relevant for the tenderer to know it. Suppose a tenderer knows that he has produced far more "green electricity" in the past two years than his competitors – then if he knows that the weighting is strongly in his favor, he might be tempted to offer a higher price than if he is insecure about the exact value of the advantage he has. It is plausible that the European Court of Justice had such a situation in mind when it judged in the ATI case that it is admissible to keep those parts of the award system secret that are irrelevant for the preparation of the tenders. If so, that judgment is very wise, keeping a perfect balance between the principle of transparency and economic interests.

The example given is incomplete, since it is impossible to publish the weightings of the criterion and the second subheading (as is required according to the ATI-judgment) and at the same time keep the weighting of the first subheading secret. So there needs to be another subheading with irrelevant weighting, for example, the quality of the measures that have been taken in the last two years to protect bird life threatened by windmills, which are used for the production of electricity from renewable energy sources.

If in the contract documents it is published that the criterion of 'green electricity' has a weighting of 30 points and the relevant subheading 2 has a weighting of 10, the secret irrelevant weightings

of subheadings 1 and 3 could be any combination like 19 + 1, 18 + 2, 17 + 3, and so on until 1 + 19 points. Although in Dutch case law there is an example where the Court of Appeal accepted that the weighting of a subheading was zero, this judgment seems to be 'one step too far' if we compare it with the ATI-judgment which is of a later date.[8] If we look at these numbers, it is obvious that this secret specification must be made before the contracting authority has opened the envelopes, or – even better – before the tenders are submitted.

APPLYING ECONOMIC METHODS TO DISCOUNT

Sometimes the contracting authority asks the tenderers to offer a discount, for example if the yearly turnover under the contract exceeds a certain level. In software maintenance contracts, after an initial period the services will be provided with more efficiency because the staff has become familiar with the software and because usually the number of incidents to be solved is decreasing. In such contracts a discount of the price after the initial period is common. Let us consider a procurement of a software maintenance contract where in the contract documents a discount as a percentage of the monthly price is asked after the first year and there are points given for the discount offered. If the award system is fully transparent, the optimal value of the discount can be calculated by the tenderer. An example of this calculation is given in Table 4.

With a discount of 0%, the total cost of ownership (TCO) is €100 + €100 + €100 = €300. In case a discount is given, the tenderer will raise the price accordingly in order to reach the same turnover. The higher the discount, the higher the price for the first year will be and this can be calculated with mathematical precision. For example, if

TABLE 4
Calculation of the Optimal Value of Discount

Price 1st year	Score	Discount	Score	Total score	TCO
€100	50.00	0%	0	50.00	€300
€100.67	49.33	1%	0.7	50.03	€300
€101.35	48.65	2%	1.4	50.05	€300
€102.04	47.96	3%	2.1	50.06	€300
€102.74	47.26	4%	2.8	50.06	€300
€103.45	46.55	5%	3.5	50.05	€300

the discount is 4% and the price for the first year is P, the calculation is:

P + 0.96P + 0.96P = €300, or

P(1 + 0.96 + 0.96) = €300, so

P = €300 / (1 + 0.96 + 0.96) = €102.74.

The formula used for the calculation of the score for price is linear, Score = 50 – (P – 100) points, and also the formula for the score of discount is linear: Score = 0.7D points. Since the sequence of prices is not linear (the numbers grow faster than linear) as shown in Table 4, there is an optimal value for the discount, which is approximately 3.6%. With the formulas given, a precise mathematical calculation is possible. This is left as an exercise for the reader who is still familiar with school mathematics.

Although this award system seems to lead to an acceptable result, it is not a good system, because the optimal value depends on the relative weightings of price and discount, which usually cannot be determined beforehand. Let's see what happens if the weighting of discount is too low (Table 5).

TABLE 5
Weighting of Discount Too Low

Price 1st year	Score	Discount	Score	Total Score	TCO*
€100	50.00	0	0	50.00	€300
€100.67	49.33	1%	0.5	49.83	€300
€101.35	48.65	2%	1.0	49.65	€300
€102.04	47.96	3%	1.5	49.46	€300
€102.74	47.26	4%	2.0	49.26	€300
€103.45	46.55	5%	2.5	49.05	€300

Note: * TCO = Total cost of ownership.

The formula for the score of discount has been changed to Score = 0.5D. Obviously, no discount will be offered with a weighting that is so small that the score will decrease if any discount is offered. Sometimes this can be acceptable, but if the weighting of discount is too high, a situation arises that will nearly always be unacceptable, as is shown in Table 6.

TABLE 6
Weighting of Discount too high

Price 1st year	Score	Discount	Score	Total Score	TCO
€100	50.00	0	0	50.00	€300
€107.14	42.86	10%	12.5	55.36	€300
€115.38	34.62	20%	25.0	59.62	€300
€125.00	25.00	30%	37.5	62.50	€300
€136.36	13.64	40%	50.0	63.64	€300
€150.00	0.00	50%	62.5	62.50	€300

Note: * TCO = Total cost of ownership.

Now the formula for the score of discount has been changed to Score = 1.25D (the values of discount shown are 10 times larger than in the former two tables). The optimal value is reached at a value of approximately 40%, which will be unacceptably high. With a larger weighting of discount it is possible that the optimal value reaches 100%. In that case, the services will be delivered free of charge in the second and third year, thus the TCO of €300 will be all paid in the first year. Clearly this is unacceptable as after the first year there will be no stimulus for the performance of the supplier – usually the possibility of a termination of the contract by the contracting authority will force the supplier to maintain at least a certain minimum level of quality.

What we have learned from this example is that if a separate score for discount is given, it is strongly advised to verify that under no circumstances the weighting of discount is too high. If it is too low, not much harm is done, so probably it is better to stay on the safe side with a relatively small weighting.

GAME THEORY METHODS

In the last paragraph the award system was fully transparent – each tenderer could calculate his score beforehand, without knowing what the other tenderers will do. But such systems are rarely used in practice, in most procurements a system is used where some score for a criterion – or even all scores – are relative. Especially the score for price is in many systems calculated by comparison with the lowest price, or with the highest price or both. We have seen that in such systems the rule of Independence of Irrelevant Alternatives is violated

and a ranking paradox is possible. This paragraph will show that the tenderer can only find his optimal tender with game theory methods.

Like in game theory the tenderers can be seen as players who will have to find their best possible strategy by calculating or guessing what the other players will do. Game theory can be applied to many fields of law, for example in the case where a victim wants to recover damages from an injurer. Under certain conditions it is possible to predict whether or not the victim and the injurer will reach a settlement before trial (Baird, Gerner & Picker, 1994).

Let us see how game theory works in the example of the calculation of the optimal discount in an award system where the score for discount depends on the other tenders. Suppose that the tender offering the highest discount will receive 4 points and the other tenders will get Score = 4 × D / H, where H = highest discount and D = discount offered. So if H = 4%, tender A offering a 2% discount will get 2 points. Let us for simplicity presume that a discount can be compensated for by raising the price with the same percentage and that the tenderer can estimate that 1 point is subtracted for each percent that the price is higher. Now, if we presume that H equals 3%, the calculation can be made as shown in Table 7.

Under the assumptions made, the weighting of discount is high enough to make it profitable to offer a 3% discount. The tenderer may even consider 4% - in doing so, he will not improve his own score but he will cause the scores of all other tenders (except those offering 0%) to decrease. It can easily be seen that if tenderer A assumes H to be 5%, he would be wise to offer 0%. By the way, if the score for discount is calculated like in the example, and one has estimated that the best choice is to offer 0% discount, it is a good strategy to

TABLE 7
Finding the Optimal Discount with H = 3%

Price/Discount Combinations	Price	Price Score	Discount	Discount Score	Total Score
Option 1	€100	50	0%	0	50
Option 2	€101	49	1%	1.33	50.33
Option 3	€102	48	2%	2.67	50.67
Option 4	€103	47	3%	4	51

offer 0.1% discount instead of 0%, just in case all other tenders offer 0%. If the latter is the case, offering 0.1% is a very good bargain – the maximum score at hardly any costs! It will be clear that in this award system, the tenderer will face a difficult decision, known in game theory as the *Prisoner's Dilemma.*

Applying Game Theory to Price Criteria

In public procurement, quite often several prices have to be submitted. For instance in a services contract with a duration of 2 years, the services can be delivered cheaper in the second year than in the first year because there are no installation costs. So it looks natural to ask for two separate prices and to give separate scores for each of them. The following example will show that this is a very risky award system. Suppose that the scores are calculated with the formulas:

$S_1 = 30 \times L_1 / P_1$, and

$S_2 = 20 \times L_2 / P_2$

Where:
L_1 = lowest price for the first year,
P_1 = price for the first year for which the score is calculated, etc.

Let us assume that tenderer A has calculated his prices according to the standard policy of his company as $P_1 = €60$ and $P_2 = €40$. Without taking much risk, A might consider the possibility to bid the strategic prices of $P_1 = €55$ and $P_2 = €45$ instead of the calculated prices. This would yield the same turnover for him, neglecting the fact that part of the price is paid a year later. How the second option works out, can only be calculated if the lowest prices are known. Suppose that A assumes the following values: $L_1 = €50$ and $L_2 = €30$. The scores of the two options for A are shown in Table 8.

TABLE 8
Strategic Bidding with Two Prices

Tender	Price 1st year	Score	Price 2nd year	Score	Total score
A – option 1	€60	25	€40	15	40
A – option 2	€55	27.3	€45	13.3	40.6
B	€50	30	€48	12.5	42.5
C	€65	23.1	€30	20	43.1

It is strange that in the award system it is possible to improve one's score by simply moving part of the price from the first year to the second year. It can be calculated that if L_1 and L_2 are low, option 2 will always be better than option 1 if $L_2 < 0.818 \times L_1$. If the other tenderers bid 'normal' prices, A can win the procurement with almost 100% certainty by bidding $P_1 = 0$ and $P_2 = 100$, as is shown in Table 9.

TABLE 9
The effect of bidding a price of zero

Tender	Price 1st year	Score	Price 2nd year	Score	Total score
A – option 3	€0	30	€100	6	36
B	€50	0	€48	12.5	12.5
C	€65	0	€30	20	20

Bidding a price of zero for a part of the services has a disastrous effect on the scores of the other tenders. It is a very risky strategy, since almost certainly the contracting authority will be aware of the fact that it pays too much (the TCO of both B and C is smaller than the TCO of A). If the contracting authority has the possibility of making a choice between the services, e.g. if in the example it can terminate the contract for convenience after the first year, the strategy is too risky. But the strategy to bid option 3 instead of option 1 has been applied successfully several times. For example, in three different cases prices of zero or €0.01 for certain services or items have been submitted by several tenderers. In each award system involved in those cases, the formula for the score of price had the lowest price in the denominator, thus implying a division by zero, which is impossible. In two cases because of this impossibility to apply the formula, the tender with a price of zero was declared invalid by the court.[9] In the third case, the President of the Court of First Instance argued that '[...] the Commission disregarded the technical specification and committed a manifest error of assessment by accepting the absence of any indication of a price or the indication of a zero price for certain items [...]'.[10] However, on other grounds in this case the application for interim measures was dismissed by the President.

WILL AWARD SYSTEMS BE CHANGED?

In two articles in a Dutch magazine the author has argued that relative scores should be avoided since they bear the risk of a ranking paradox and make an economic optimization impossible (Chen, 2005, 2006). Since their publication there are hardly any changes visible in the award systems used by contracting authorities in The Netherlands. In a recently published third article, the possibility of strategic bidding with game theory methods is described (Chen, 2007). Other authors have recently warned that many award systems used in practice are risky (Meijer & Telgen, 2007). Maybe the awareness of the risks that strategic bidding may bring to them will cause contracting authorities to use fully transparent award systems where an economic optimization will yield the truly most economically advantageous tender.

CONCLUSIONS

As the European Procurement Directives mention the criterion of the most economically advantageous tender, it would be normal for tenderers to use economics methods to arrive at their optimum proposal. It can be argued that the point of view that the Directives require a full transparency in this economic sense is supported by the case law of the European Court of Justice. The reality, however, is that many award systems are not fully transparent because of the use of relative scores, which do not allow for application of economic methods. This leaves tenderers no other option than to apply game theory methods. Unfortunately, those methods usually will not yield the 'best possible proposal' for the contracting authority. There is a solution for this problem – although it requires some knowledge of elementary mathematics, designing a good award system should in most cases not be a big problem.

NOTES

1. Court of Appeal of Den Bosch 17 July 2007, LJN: BB1674.

2. Interlocutory Proceedings Court of Leeuwarden 21 January 2005, LJN: AS3521, Interlocutory Proceedings Court of Haarlem 14 July 2006, LJN: AY3951.

3. European Court of Justice 18 October 2001, C-19/00 *(SIAC)*, No. 43.

4. Interlocutory Proceedings Court of Zutphen 30 November 2007, LJN: BB9151, No. 4.7.

5. European Court of Justice 24 January 2008, case C-532/06 *(Lianakis),* No. 45.

6. European Court of Justice 24 November 2005, case C-331/04 *(ATI)*, No. 32*.*

7. European Court of Justice 4 December 2003, case C-448/01 *(Wienstrom).*

8. Court of Appeal of The Hague 4 August 2005, LJN: AU0544.

9. Interlocutory Proceedings Court of Utrecht 13 June 2007, LJN: BA7015 and Interlocutory Proceedings Court of Groningen 3 August 2007, LJN: BB2292.

10. President of the Court of First Instance 31 January 2005, case T-447/04 R *(Capgemini)*, No. 82.

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